

A Theory of Turnover and Wage Dynamics*

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Abstract

We develop a model of turnover and wage dynamics with insurance, match-specific productivity, and long-term contracting. The model predicts that wages are downward rigid within firms but can decrease when workers are fired. We apply the model to study the impact of business cycles on subsequent wages and job mobility. Workers hired during a boom have persistent higher future wages if staying with the same firm. However, these boom hires are more likely to be terminated and have shorter employment spells.

Keywords: long-term contracts, business cycles, match-specific productivity

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1 Introduction

Growing evidence has suggested that random economic shocks at the time of employment have large and persistent effects on subsequent labor market outcomes. Baker, Gibbs, and Holmstrom (1994) use personnel records from one firm and identify a *cohort effect*—a cohort’s average wage at the start of employment affects its wages years later. Subsequent within-firm studies (see Gibbs and Hendricks (2004) for a review) have also documented the cohort effect within the same employment relationship. Beaudry and DiNardo (1991) document the cohort effect in large, representative data sets, and they show that the cohort effect is driven by the lowest unemployment rate since the worker was hired.

Random economic shocks also affect long-term earnings by changing the types of jobs workers choose, and its effects are particularly strong among workers fresh out of school. Oyer (2008) shows that MBAs graduating in recession years are significantly less likely go to Wall Street, and consequently, suffer an estimated lifetime-earnings loss of \$1.5 to \$5 million. Oreopoulos, von Wachter, and Heisz (2012) find that a typical recession causes Canadian college graduates to take lower-quality jobs, and as a result, the cumulated earnings in the first ten years of their career are reduced by about five percent. Kahn (2010) shows that a bad economy leads U.S. college graduates to choose less-prestigious occupations, and its negative effects on wages persist even after 15 years.

Despite its negative effects on long-term earnings, a bad economy appears to have a positive effect on job security. Schmieder and von Wachter (2010) use the Displaced Worker Survey and find that the probability of job loss is smaller for workers who were hired in worse labor market conditions. Kahn (2008) exploits a proprietary dataset for large U.S. firms and finds that while jobs created during a recession typically do not last as long, this is mainly because jobs with shorter durations are more likely to be offered in a recession. Once the job heterogeneity is controlled for, workers hired in a recession have longer expected employment spell.

The positive effect on job security is inconsistent with existing theories that explain the negative effect of a recession on long-term earnings. Schmieder and von Wachter (2010) discuss three classes of models on earnings, including human capital accumulation, job matches, and insurance. They note that models based on human capital accumulation and job matches have the opposite prediction on job security. In particular, if workers hired in a bad economy have lower wages because they are less productive, either through worse firm-specific or task-specific human capital accumulation opportunities or because of worse match qualities, they should have worse rather than better job security.¹ In models based on insurance, which we will discuss below, worker productivity is assumed to be the same across firms, so there is no turnover.

In this paper, we develop a simple model that integrates insurance, match-specific productivity, and long-term contracts. When match-specific productivity is important, we show that a bad

¹See Gibbons and Waldman (2004, 2006) for the theory of task-specific human capital.

economy lowers long-term earnings, but the workers hired in a bad economy have better employment security. By integrating both insurance and match-specific productivity, our model inherits the properties of insurance models on wage dynamics: wages are downward rigid within an employment relationship. Having match-specific productivity, however, implies that bad matches can be terminated, so turnover does occur. As will be explained in detail in Section 4, the key mechanism of the model is that there is less scope for providing insurance to a worker in a bad economy, so the value of starting an employment relationship is lower. This implies that workers in a bad economy need to be better matches to be hired. Consequently, the average match quality of workers hired in a worse economy is better, resulting in longer employment spells.

Both insurance and match-specific productivity are familiar and important ideas in labor economics. Harris and Holmstrom (1982) show that many features of the labor market can be explained by an insurance model in which firms can commit to long-term contracts for risk-averse workers who are free to move. Beaudry and DiNardo (1991) find that the Harris-Holmstrom model offers a better description of wage dynamics than does a spot market model or a full insurance model. However, these insurance models do not generate turnover because the productivity of the workers is identical across all firms, making turnover meaningless.

For turnover patterns, a leading model is Jovanovic (1979), which is based on learning and match-specific productivity. Its celebrated prediction, that turnover probability is inverse-U shaped with job tenure, has been confirmed by empirical findings; see, for example, Farber (1994). In Jovanovic (1979), the initial expected match quality is the same for all hired workers. Here, in contrast, we assume that before hiring a worker, the firm observes a signal about his match-specific productivity. This allows the firm to base its hiring decision on the signal observed and also allows the average match quality of the hired workers to differ with the economic condition at the time of hiring.

The rest of the paper is organized as follows. We set up the model in Section 2. Section 3 describes the main features of the optimal employment contract, which are then used to study the effects of random economic shocks on subsequent labor market outcomes in Section 4. Section 5 discusses how the results of the paper change with alternative formulations. Section 6 concludes.

2 Setup

We consider a labor market with a large number of identical risk-neutral firms and risk-averse workers. Firms produce a single consumption good with labor as the only input. Both firms and workers live for infinite number of periods with $\delta \in (0, 1)$ as the discount factor. There are two possible states of the economy: boom and bust. We denote θ_L as the general productivity level in a bust and θ_H as the general productivity level in a boom, with $\theta_H > \theta_L > 0$. In each period, the probability of a boom state is $p \in (0, 1)$. The state of the economy is assumed to be i.i.d. across

periods. Let θ_t , $t = 0, 1, 2, \dots$ be the general productivity in period t . The state of the economy is public information and becomes known at the beginning of each period.

Each firm hires at most one worker in each period. Firm's output in period t is given by

$$y_t = \theta_t + \tilde{m},$$

where $\theta_t \in \{\theta_L, \theta_H\}$ is the state of economy and $\tilde{m} \in \{-m_b, m_g\}$ is the match-specific quality between the firm and the worker, with $m_g > 0$ denoting a good match and $-m_b < 0$ denoting a bad match.

The match quality is not known before the worker works for the firm. However, after a worker works for a firm, the output is realized at the end of each period. Since the worker's match quality is equal to his output minus the general productivity level in each period, the exact match quality is thus fully revealed at the end of the first hiring period. In particular, at the beginning of the initial period, both parties observe a signal $\alpha \in [0, 1]$, which denotes the probability that the worker is a good match. We assume that α has a non-zero density function f with support in $[0, 1]$.

We assume that firms have the full bargaining power in the labor market, and we relax this assumption in Section 5. Specifically, upon observing the signal, the firm decides whether to make an offer to the worker. If no offer is made, both parties receive their outside options. If the firm makes an offer, the employment contract should specify the wages and retention rule for the current and each of the subsequent periods. Since the match quality becomes public knowledge at the end of the first hiring period, the wages and hiring rule for the subsequent periods should depend on the match type. Formally, an employment contract is represented by $\{w_0, w_g(\theta^t), w_b(\theta^t), h_g(\theta^t), h_b(\theta^t)\}_{t=1}^\infty$, where w_0 is the worker's wage in the initial period, $w_g(\theta^t)$ (or $w_b(\theta^t)$, respectively) is the worker's wage in period t in case of a good (or bad, respectively) match, given that the history of the states of economy is $\theta^t = (\theta_0, \theta_1, \dots, \theta_t)$, and $h_g(\theta^t)$ (or $h_b(\theta^t)$, respectively) is the probability that the firm retains the worker in period t , given the state history θ^t and that the match quality is good (or bad, respectively). Note that letter "g" which appears in the subscripts stands for a "good match" and "b" stands for a "bad match".

We assume that only firms are able to commit to the employment contracts. Workers may choose to quit and meet new firms elsewhere without any financial obligation to the original firm. In other words, a firm will offer a contract as long as it yields an expected payoff at least as high as its outside option at the beginning of the initial period. Once the contract is accepted, the firm will stick to it no matter what happens in the future. On the other hand, to prevent the worker from quitting when he is not laid off, the expected continuation payoff yielded by the contract should be no lower than the worker's outside options for all subsequent periods.

To characterize each party's payoffs, assume that in the periods when a worker is hired by a firm, the worker receives his wage, as described in the contract, and the firm gets the output net of

the wage. In case of a separation between a worker and a firm, both parties receive their outside options. A separation takes place when (i) the firm decides not to offer a contract in the initial period; (ii) the worker rejects the contract in the initial period; (iii) the worker gets fired in some future period according to the contract terms; (iv) an exogenous separation occurs at the beginning of each period, with a separation rate $1 - k \in [0, 1]$; or (v) the worker decides to quit in some future period.

In particular, if a separation takes place at the beginning of period t , with θ_t as the state of the economy, then for the current period t , the firm receives nothing and the worker earns θ_t . The interpretation is that the worker can engage in home production with the general productivity θ_t in that period. After that, both the worker and the firm re-enter the labor market in period $t + 1$. Thus, firms' outside option in case of a separation, $\underline{\Pi}$, does not depend on the states; and the worker's outside option is a function of the state of the economy, $\underline{U}(\theta)$, with $\theta \in \{\theta_H, \theta_L\}$.

Note that since firms have the full bargaining power, an optimal employment contract always leaves the worker exactly with his outside option in the initial period. We must have

$$\begin{cases} \underline{U}(\theta_H) = u(\theta_H) + \delta[p\underline{U}(\theta_H) + (1 - p)\underline{U}(\theta_L)] \\ \underline{U}(\theta_L) = u(\theta_L) + \delta[p\underline{U}(\theta_H) + (1 - p)\underline{U}(\theta_L)] \end{cases},$$

or

$$\begin{cases} \underline{U}(\theta_H) = u(\theta_H) + \frac{\delta}{1-\delta} [pu(\theta_H) + (1 - p)u(\theta_L)] \\ \underline{U}(\theta_L) = u(\theta_L) + \frac{\delta}{1-\delta} [pu(\theta_H) + (1 - p)u(\theta_L)] \end{cases}. \quad (1)$$

In other words, a worker receives his outside option when he conducts home production with the general productivity in each period.

Finally, we assume that while firms are risk-neutral, workers are risk-averse with a VNM utility function $u(\cdot)$, which is twice-differentiable, increasing and strictly concave.

3 The Optimal Contracts

In this section, we study the optimal employment contracts. Let $\Pi(\alpha, \theta_0)$ be the firm's expected payoff yielded by the optimal contract, given that the initial state of the economy is $\theta_0 \in \{\theta_H, \theta_L\}$ and the signal of match quality is $\alpha \in [0, 1]$. Thus, we have the following program.

$$\begin{aligned} \Pi(\alpha, \theta_0) &= \max_{\{w_0, w_g(\theta^t), w_b(\theta^t), h_g(\theta^t), h_b(\theta^t)\}_{t=1}^{\infty}} \{\alpha m_g - (1 - \alpha)m_b + \theta_0 - w_0 \\ &+ \alpha \sum_{t=1}^{\infty} \delta^t E_{\theta^t} [\sigma_g(\theta^t) (m_g + \theta_t - w_g(\theta^t)) + (\sigma_g(\theta^{t-1}) - \sigma_g(\theta^t)) \underline{\Pi}] \\ &+ (1 - \alpha) \sum_{t=1}^{\infty} \delta^t E_{\theta^t} [\sigma_b(\theta^t) (-m_b + \theta_t - w_b(\theta^t)) + (\sigma_b(\theta^{t-1}) - \sigma_b(\theta^t)) \underline{\Pi}] \end{aligned}$$

subject to

$$\begin{aligned}
& u(w_0) + \alpha \sum_{t=1}^{\infty} \delta^t E_{\theta^t} [\sigma_g(\theta^t) u(w_g(\theta^t)) + (\sigma_g(\theta^{t-1}) - \sigma_g(\theta^t)) \underline{U}(\theta_t)] \\
& + (1 - \alpha) \sum_{t=1}^{\infty} \delta^t E_{\theta^t} [\sigma_b(\theta^t) u(w_b(\theta^t)) + (\sigma_b(\theta^{t-1}) - \sigma_b(\theta^t)) \underline{U}(\theta_t)] \\
& \geq \underline{U}(\theta_0); \tag{IR}
\end{aligned}$$

$$\begin{aligned}
& u(w_g(\theta^t)) + \sum_{j=t+1}^{\infty} \delta^{j-t} E_{\theta^j | \theta^t} \left[\frac{\sigma_g(\theta^j)}{\sigma_g(\theta^t)} u(w_g(\theta^j)) + \left(\frac{\sigma_g(\theta^{j-1})}{\sigma_g(\theta^t)} - \frac{\sigma_g(\theta^j)}{\sigma_g(\theta^t)} \right) \underline{U}(\theta_j) \right] \\
& \geq \underline{U}(\theta_t), \text{ if } \sigma_g(\theta^t) > 0; \tag{IC}_g
\end{aligned}$$

$$\begin{aligned}
& u(w_b(\theta^t)) + \sum_{j=t+1}^{\infty} \delta^{j-t} E_{\theta^j | \theta^t} \left[\frac{\sigma_b(\theta^j)}{\sigma_b(\theta^t)} u(w_b(\theta^j)) + \left(\frac{\sigma_b(\theta^{j-1})}{\sigma_b(\theta^t)} - \frac{\sigma_b(\theta^j)}{\sigma_b(\theta^t)} \right) \underline{U}(\theta_j) \right] \\
& \geq \underline{U}(\theta_t), \text{ if } \sigma_b(\theta^t) > 0; \tag{IC}_b
\end{aligned}$$

$$0 \leq h_g(\theta^t), h_b(\theta^t) \leq 1, \text{ for any } t \text{ and } \theta^t.$$

Here $\sigma_g(\theta^t) \equiv k^t \Pi_{i=1}^t h_g(\theta^i)$ is defined to be the probability that the good-match worker is still hired in period t , given the history of states of the economy θ^t ; and $\sigma_b(\theta^t) \equiv k^t \Pi_{i=1}^t h_b(\theta^i)$ is the correspondence for the bad-match worker². Thus, for each $s = g$ or b , $\sigma_s(\theta^{t-1}) - \sigma_s(\theta^t)$ is the probability that the s -type worker gets fired at the beginning of period t , given the history of states θ^t .

With the definitions of $\sigma_g(\theta^t)$ and $\sigma_b(\theta^t)$, the objective function of the above program gives the firm's total expected payoff and the left-hand side of IR gives the worker's total expected utility.

The IR constraint simply states that the worker's expected utility at the time when the contract is signed is not below its outside option. Note that IR should be binding in the optimal contract because if it were slack, the firm can increase its payoff by decreasing the worker's initial wage w_0 . Constraints IC_g and IC_b state that for any match quality or state of the economy, as long as the worker is not laid off, staying with the firm always gives the worker an expected continuation utility at least as high as his outside option. Thus, the worker will never quit an employment contract unless the contract fires him or an exogenous separation occurs.

²We assume that $\sigma_s(\theta^0) = 1$ for each $s = g$ or b and any $\theta^0 \in \{\theta_H, \theta_L\}$.

Finally, the firm's outside option is given by

$$\underline{\Pi} = \delta E_{\theta, \alpha} \max \{ \underline{\Pi}, \Pi(\alpha, \theta) \}. \quad (2)$$

3.1 The Optimal Retention Rule

In this subsection, we discuss the properties of turnover in an optimal contract. Our first result shows that once revealed to be a good match, the worker should never be laid off.

Proposition 1 *The optimal hiring rule is that $h_g(\theta^t) = 1$, for all $t \geq 1$ and all θ^t .*

Proof. Suppose the optimal contract fires a good-match worker in period t . Then the continuation payoffs at the beginning of period t are $\underline{\Pi}$ for the firm and $\underline{U}(\theta_t)$ for the worker, where θ_t is the state of the economy in period t . Now consider a deviation in which (i) the worker is kept in period t and paid a wage θ_t , and (ii) the contract from period $t + 1$ on is specified as if the worker and the firm met in period $t + 1$ with a signal of match quality $\alpha = 1$.

Obviously, the worker is indifferent between the above deviation and the original contract, because the worker's expected continuation payoff in period t remains $\underline{U}(\theta_t)$. On the other hand, note that the firm receives m_g instead of zero in period t , and receives $\Pi(1, \theta_{t+1})$ in period $t + 1$. Thus, its expected continuation payoffs at the beginning of period t becomes $m_g + \delta E_{\theta} \Pi(1, \theta) > \underline{\Pi}$ according to (2), implying that the deviation makes the firm strictly better off, which completes our proof. ■

The intuition for Proposition 1 is that keeping a good-match worker not only results in a higher output in the current period, but also keeps the firm in the best situation in the future because the worker it hires in the next period is a good match for sure, which may not be the case when the firm meets a new worker elsewhere.

While a good-match worker should always be retained, termination occurs in our model because the worker's productivity depends on his match quality. Terminating a bad match therefore has the benefit of increasing the value of the relationship through higher productive efficiency. This contrasts with Harris and Holmstrom (1982) and Beaudry and DiNardo (1991) in which no termination occurs because the worker's productivity is the same in all firms.

For the bad match, the retention rule compares the benefit of production efficiency with the cost of loss of insurance. On the one hand, terminating a bad match is beneficial from a joint-production standpoint since the worker is more productive with his outside option. On the other hand, terminating the worker is costly because it exposes the worker with the risk of a lower wage³. Note that reducing the risk of the worker benefits the firm by allowing it to lower the worker's

³We will show later that if the worker is retained by the firm, his wage in an optimal contract never falls.

initial wage. Thus, the optimal retention rule in case of a bad match reflects the trade-off between avoiding production inefficiency and providing employment insurance to the worker.

Throughout this paper, we assume that match quality is sufficiently important, that is, m_b is sufficiently large, so that the marginal benefit of offering more employment insurance—in the form of lowering the probability that the bad match is terminated—falls short of the loss of keeping the bad match. In other words, the optimal contract always terminates the bad match after the initial period. We make this assumption mostly for analytical simplicity so that the ex post threshold of termination does not depend on the ex ante timing of hiring. The plausibility of the assumption clearly depends on the conditions of the labor market and the types of jobs considered. The assumption is more likely to be satisfied when the skill requirements for the job are complex and the market for such jobs is thin. Match quality is likely to be more important, for example, when the job requires a large combination of different skills.

3.2 Properties of Wage Dynamics

Now we discuss the wage dynamics of the optimal contract. The first lemma in this subsection states that in the optimal contract, the worker’s wage is downward rigid.

Lemma 1 *For any $t \geq 1$ and any history of the states of the economy θ^t , we have*

$$w_g(\theta^t, \theta_{t+1}) \geq w_g(\theta^t), \text{ for each } \theta_{t+1} \in \{\theta_H, \theta_L\}.$$

Proof. See Appendix. ■

The reason for the downward rigidity is the same as in Harris and Holmstrom (1982). In particular, consider a worker retained by the firm in period $t + 1$. Suppose he experiences a wage loss, the firm can then smooth the worker’s pay by raising the period $t + 1$ wage and lowering the period t wage. This change generates a larger value for the relationship since it offers better insurance to the worker. Since the worker has the full bargaining power in the employment relationship, he can get strictly better off through such changes. This implies that the wage must be downward rigid in the optimal contract. While being downward rigid, the wage is not upward rigid. Whenever the retained worker’s outside option exceeds his continuation expected payoff as if he was paid the current wage forever, there will be a wage jump in the next period to match it. This lack of full insurance arises because the worker cannot commit to the contract. The worker’s opportunity to take the outside option therefore limits the scope of insurance.

It is clear that the wage jumps can never take place at bust states. A wage jump at a bust state implies that the worker’s expected continuation payoff before the jump should be strictly lower than $\underline{U}(\theta_L)$, which is never possible due to constraint (IR). On the other hand, while the

wage jumps occur at boom states, it happens at most once. This is because the worker has the highest outside option at a boom state. Once the wage jumps at a boom state to meet the outside option, the worker will never take his outside options in the future. Thus, from that period on, the optimal contract can offer full insurance by paying the worker the same wage forever, as long as he is retained by the firm. Proposition 2 below describes the optimal wages as functions of the state of the economy.

Proposition 2 *The optimal wages depend on the initial state of the economy.*

(a) *Starting with a boom state, the optimal contract offers an initial wage $w_0(\alpha, \theta_H)$ that solves*

$$\begin{aligned} & u(w_0(\alpha, \theta_H)) + (1 - \alpha) \delta \underline{EU} \\ & + \alpha \sum_{t=1}^{\infty} [\delta^t k^t u(w_0(\alpha, \theta_H)) + (\delta^{t-1} k^{t-1} - \delta^t k^t) \underline{EU}] \\ & = \underline{U}(\theta_H). \end{aligned} \tag{3}$$

Moreover, the retained worker is paid $w_0(\alpha, \theta_H)$ for all the subsequent periods, regardless of the states of the economy.

(b) *Starting with a bust state, the optimal contract pays an initial wage $w_0(\alpha, \theta_L) = \theta_L$. In addition, the optimal wage keeps being θ_L until the first boom state, at which the wage jumps to w_H and never changes in the future, where w_H solves*

$$u(w_H) + \delta [k \underline{U}(\theta_H) + (1 - k) \underline{EU}] = \underline{U}(\theta_H) \tag{4}$$

where $\underline{EU} = p \underline{U}(\theta_H) + (1 - p) \underline{U}(\theta_L)$.

Proof. See Appendix. ■

Proposition 2 shows that the optimal contract signed at a boom state offers full insurance on the worker's wages; and the optimal contract signed at a bust state offers only partial insurance. In particular, starting with a bust state, the optimal contract offers a low initial wage θ_L , and the wage is forced to jump to w_H at the first boom state in the future to meet the worker's higher outside option. It follows from (4) that w_H is the wage level at which the worker's expected continuation payoff is exactly equal his outside option at a boom state.

The key property of the optimal employment contract is that employment relationships starting in the boom state provide more insurance. To see this, note that the worker's outside option is higher in a boom state, so there is more scope for insurance. In particular, it is more attractive in a boom state for the worker to accept a lower starting wage (relative to his outside productivity) for future insurance. This lower cost of providing insurance implies that the value of the employment relationship is higher in a boom state, making the parties more likely to start a relationship. The next section uses this observation to explore the effect of business cycles on employment dynamics.

4 Implications of Business Cycles on Turnover and Wages

In this section, we apply the properties of the employment contract derived in the previous section to study its implications on hiring, wages, and subsequent turnover over the business cycle.

To study the impact of the business cycle on the hiring threshold, recall that α is the firm's signal about the probability of the worker being a good match. It is clear the expected value of the relationship is increasing in α , so the firm makes an offer to the worker if and only if the signal is above a threshold. Denote α_H and α_L as the hiring thresholds for the boom and bust states, respectively. An immediate consequence of Proposition 2 is that the hiring threshold is lower in a boom state.

Lemma 2 *The marginal worker hired in a boom state has a lower expected match quality than the marginal worker hired in a bust state:*

$$\alpha_H < \alpha_L.$$

Proof. See Appendix. ■

The reason for this was alluded to at the end of the previous section—the value of starting an employment relationship is larger in a boom state than in a bust state. The extra value results from the additional insurance the firm can provide to the worker hired in a boom. In specific, with the threshold signal α_L , the optimal contract made at a boom state yields a strictly higher expected payoff to the firm than its outside option. This implies that the marginal signal α_H in a boom state is strictly lower than α_L .

Lemma 2 provides one reason for the variations of job-finding rates during the business cycle. Hall (2005) reports that the increase in the unemployment rate during the recession is driven largely by changes in job-finding rates instead of job-separation rates. This model shows that job-finding rates can drop in recessions because firms become "pickier" by raising the bar in terms of match quality. Evidence from the popular press suggests that this mechanism may be at work. A 2011 Wall Street Journal article quotes Paul Marchand, head of talent acquisition at a New York food-and-beverage company, as saying, "People think that with all the available talent, time-to-fill would go down, but it's just the opposite. When you're still trying to find quality candidates, it's actually taken longer."⁴ Such emphasis on "quality candidates" suggests that companies are looking for better matches during the recession.

The difference in the hiring thresholds implies that the initial labor market condition affects the composition of the workers hired, which directly affects the subsequent patterns of wages and turnover. The next proposition reports a few empirically testable predictions on the impact of

⁴Joe Light, "Jobs Open, but Filling Them Slows Down," Wall Street Journal, March 7, 2011.

business cycles. The empirical literature has distinguished between voluntary and involuntary turnover. To connect our results to the empirical findings, we define turnover as voluntary if the worker's new wage is strictly higher than his previous wage.⁵ Otherwise, turnover is involuntary. There are other ways to distinguish voluntary and involuntary turnover in this model. For example, we can define that a worker leaves involuntarily if $h = 0$, and it yields the same qualitative result. The current definition is chosen because it depends on variables that are empirically observable.

Proposition 3 *The initial state of the economy has persistent effects on wages and turnover.*

(a) *A worker hired in a boom has a higher wage in all subsequent periods if he stays with the same firm. But the initial state has no effect if the worker leaves. That is, for any $t \geq 1$,*

$$\begin{aligned} E_{\alpha, \theta^t}[w_g(\theta^t)|\alpha \geq \alpha_H, \theta_0 = \theta_H] &> E_{\alpha, \theta^t}[w_g(\theta^t)|\alpha \geq \alpha_L, \theta_0 = \theta_L]; \\ E_{\alpha, \theta^t}[w_b(\theta^t)|\alpha \geq \alpha_H, \theta_0 = \theta_H] &= E_{\alpha, \theta^t}[w_b(\theta^t)|\alpha \geq \alpha_L, \theta_0 = \theta_L]. \end{aligned}$$

(b) *A worker hired in a boom is more likely to leave both voluntarily and involuntarily. That is,*

$$\begin{aligned} \Pr[\text{turnover with } w_b(\theta_0, \theta_1) > w_0(\alpha, \theta_H)|\alpha \geq \alpha_H, \theta_0 = \theta_H] &> \\ \Pr[\text{turnover with } w_b(\theta_0, \theta_1) > w_0(\alpha, \theta_L)|\alpha \geq \alpha_L, \theta_0 = \theta_L]; \\ \Pr[\text{turnover with } w_b(\theta_0, \theta_1) \leq w_0(\alpha, \theta_H)|\alpha \geq \alpha_H, \theta_0 = \theta_H] &> \\ \Pr[\text{turnover with } w_b(\theta_0, \theta_1) \leq w_0(\alpha, \theta_L)|\alpha \geq \alpha_L, \theta_0 = \theta_L]. \end{aligned}$$

Consequently, a worker hired in a bust has a longer average job tenure.

Proof. See Appendix. ■

Proposition 3(a) shows that the average future wage of boom hires is higher than that of bust hires. Since the average initial wage of boom hires is also higher, this is the cohort effect—a cohort's future wages are related to the initial wage upon entering the firm. As mentioned in the introduction, many empirical studies have documented the cohort effect and several theories have been put forward to explain it. The mechanism here is most related to Beaudry and DiNardo (1991). The optimal contract determines the initial wage and the future wages jointly, so both are affected by the labor market condition at the signing of the contract.

Proposition 3(a) also shows that the cohort effect, however, applies only to workers who stay with the same firm. If a worker is revealed to be a bad match and is terminated, his wage can drop, and the initial labor market condition no longer matters. There is some support for this

⁵The worker's wage in case of a turnover is assumed to be the general productivity in that period. Formally, we define that

$$w_b(\theta^t) = \theta_t, \text{ for any } t \geq 1 \text{ and } \theta^t.$$

prediction. Schmieder and von Wachter (2010) show that most of the wage gains through better labor conditions disappear after a job loss. Brunner and Kuhn (2010) find that about half of the wage gains through better labor conditions are lost after an involuntary turnover. They suggest that long-term contracts are important in generating persistent wage differences, but other factors, such as human capital accumulation, are also relevant.

Proposition 3(b) shows that a worker hired in a boom is more likely to experience both voluntary and involuntary turnover. This is because the hiring threshold is lower in a boom state, so a boom hire is more likely to be a bad match. The empirical evidence on involuntary turnover offers support for our prediction. Schmieder and von Wachter (2010) report that "workers with higher wages due to tight past labor market conditions face a higher risk of layoff." The empirical evidence on voluntary turnover, however, is mixed. Brunner and Kuhn (2010) report that for Austrian workers, "one percentage point increase in the initial employment rate decreases the probability of moving to another employer by 7.5%." In contrast, Oreopoulos, Von Wachter and Heisz (2012) find that, for Canadian college graduates, bust hires are more likely to switch jobs than boom hires. The mixed findings suggest that turnover is affected by other factors. One possibility is vertical mismatch. In particular, in a recession, fewer high-quality jobs are offered, and the new hires may overqualify for the jobs. Vertical mismatch can lead to future turnover and its importance has been studied by a number of papers (Reder 1955, Okun 1973, McLaughlin and Bils 2001, and Oyer 2006, 2008). Another related reason is the composition of the jobs offered. If high-turnover firms hire more in a recession, this leads to a higher mobility rate for the bust hires. Kahn (2008), discussed below, shows that the composition of jobs is important for understanding the effect of business cycles on turnover patterns.

Finally, Proposition 3(b) shows that bust hires have longer job tenure, which follows immediately because they are less likely to turnover both voluntarily and involuntarily. This prediction is not supported by earlier evidence. Bowlus (1995) finds a shorter employment duration for recession hires. However, Kahn (2008) points out that this shorter employment duration is driven mostly by the composition of jobs. She finds that while jobs started during a recession do on average end earlier, once firm heterogeneity is controlled for, this pattern reverses. With firm fixed effect controlled for, she shows that a one percentage increase in the national unemployment rate increases the probability that a worker stays for at least one year by five percentage points. Kahn (2008) explicitly points to match-specific productivity as the source for her findings, and this model provides a reason for why a worker hired in a recession is more likely to be a good match.

5 General Bargaining Power

We have been assuming that firms have the full bargaining power in the labor market, so that workers always earn their outside options depending on the states. In this section, we check the robustness of our results by considering exogenous bargaining power. Our key result, summarized

in Corollary 1, shows that our main results still holds even when workers have some bargaining power.

Let $\beta \in [0, 1]$ be a scale parameter that captures the exogenous bargaining power on the firms' side, with $\beta = 1$ denoting the case where the firm has the full bargaining power, and $\beta = 0$ meaning all the bargaining power goes to the worker's side. An optimal employment contract thus should maximize the weighed average of both the worker's and the firm's expected payoffs.

Let $C = \{w_0, w_g(\theta^t), w_b(\theta^t), h_g(\theta^t), h_b(\theta^t)\}_{t=1}^{\infty}$ denote an employment contract. Given the initial state of the economy $\theta_0 \in \{\theta_H, \theta_L\}$, the signal of match quality $\alpha \in [0, 1]$ and the firm's bargaining power $\beta \in [0, 1]$, under the employment contract C , the firm's expected payoff, $\Pi(\alpha, \theta_0, \beta, C)$, is given by

$$\begin{aligned} \Pi(\alpha, \theta_0, \beta, C) &= \alpha m_g - (1 - \alpha)m_b + \theta_0 - w_0 \\ &+ \alpha \sum_{t=1}^{\infty} \delta^t E_{\theta^t} [\sigma_g(\theta^t) (m_g + \theta_t - w_g(\theta^t)) + (\sigma_g(\theta^{t-1}) - \sigma_g(\theta^t)) \underline{\Pi}(\beta)] \\ &+ (1 - \alpha) \sum_{t=1}^{\infty} \delta^t E_{\theta^t} [\sigma_b(\theta^t) (-m_b + \theta_t - w_b(\theta^t)) + (\sigma_b(\theta^{t-1}) - \sigma_b(\theta^t)) \underline{\Pi}(\beta)] \end{aligned} \quad (5)$$

and the worker's expected payoff, $U(\alpha, \theta_0, \beta, C)$, is given by

$$\begin{aligned} U(\alpha, \theta_0, \beta, C) &= u(w_0) + \alpha \sum_{t=1}^{\infty} \delta^t E_{\theta^t} [\sigma_g(\theta^t) u(w_g(\theta^t)) + (\sigma_g(\theta^{t-1}) - \sigma_g(\theta^t)) \underline{U}(\theta_t, \beta)] \\ &+ (1 - \alpha) \sum_{t=1}^{\infty} \delta^t E_{\theta^t} [\sigma_b(\theta^t) u(w_b(\theta^t)) + (\sigma_b(\theta^{t-1}) - \sigma_b(\theta^t)) \underline{U}(\theta_t, \beta)], \end{aligned} \quad (6)$$

where $\sigma_s(\theta^t) \equiv k^t \prod_{i=1}^t h_s(\theta^i)$ is defined as before for each $s \in \{g, b\}$.

The optimal contract solves the following program.

$$\max_C \beta \Pi(\alpha, \theta_0, \beta, C) + (1 - \beta) U(\alpha, \theta_0, \beta, C)$$

subject to

$$\Pi(\alpha, \theta_0, \beta, C) \geq \underline{\Pi}(\beta); \quad (\text{IR}_f)$$

$$U(\alpha, \theta_0, \beta, C) \geq \underline{U}(\theta_0, \beta); \quad (\text{IR}_w)$$

$$\begin{aligned} &u(w_g(\theta^t)) + \sum_{j=t+1}^{\infty} \delta^{j-t} E_{\theta^j | \theta^t} \left[\frac{\sigma_g(\theta^j)}{\sigma_g(\theta^t)} u(w_g(\theta^j)) + \left(\frac{\sigma_g(\theta^{j-1})}{\sigma_g(\theta^t)} - \frac{\sigma_g(\theta^j)}{\sigma_g(\theta^t)} \right) \underline{U}(\theta_j, \beta) \right] \\ &\geq \underline{U}(\theta_t, \beta), \text{ if } \sigma_g(\theta^t) > 0; \end{aligned} \quad (\text{IC}'_g)$$

$$\begin{aligned}
& u(w_b(\theta^t)) + \sum_{j=t+1}^{\infty} \delta^{j-t} E_{\theta^j|\theta^t} \left[\frac{\sigma_b(\theta^j)}{\sigma_b(\theta^t)} u(w_b(\theta^j)) + \left(\frac{\sigma_b(\theta^{j-1})}{\sigma_b(\theta^t)} - \frac{\sigma_b(\theta^j)}{\sigma_b(\theta^t)} \right) \underline{U}(\theta_j, \beta) \right] \\
& \geq \underline{U}(\theta_t, \beta), \text{ if } \sigma_b(\theta^t) > 0;
\end{aligned} \tag{IC'_b}$$

$$0 \leq h_g(\theta^t), h_b(\theta^t) \leq 1, \text{ for any } t \text{ and } \theta^t. \tag{7}$$

(IR_f) and (IR_w) give the firm's and worker's individual rationality constraints, respectively, which state that the firm's and worker's expected payoffs at the time when the contract is signed should not be below their outside options. With the exogenous bargaining power, both the firm and the worker agree to start an employment relationship only when (IR_f) and (IR_w) are satisfied. Similar to the case where $\beta = 1$, constraints (IC'_g) and (IC'_b) guarantee that staying with the firm always gives the worker an expected continuation utility at least as high as his outside option, so that the worker will never quit an employment contract unless the contract fires him or an exogenous separation occurs.

Let $\Pi(\alpha, \theta, \beta)$ and $U(\alpha, \theta, \beta)$ be the firm's and worker's expected payoffs yielded by the optimal contract, respectively, given the initial state of the economy θ , the signal of match quality α and the firm's bargaining power β .⁶ Then the outside options are given by

$$\underline{\Pi}(\beta) = \delta E_{\theta, \alpha} \max \{ \underline{\Pi}(\beta), \Pi(\alpha, \theta, \beta) \}; \tag{8}$$

$$\underline{U}(\theta_H, \beta) = u(\theta_H) + \delta E_{\theta, \alpha} \max \{ \underline{U}(\theta, \beta), U(\alpha, \theta, \beta) \}; \tag{9}$$

$$\underline{U}(\theta_L, \beta) = u(\theta_L) + \delta E_{\theta, \alpha} \max \{ \underline{U}(\theta, \beta), U(\alpha, \theta, \beta) \}. \tag{10}$$

The following Lemma characterizes the optimal retention rule and wage dynamics for each $\beta \in [0, 1]$.

Lemma 3 *Regardless of the exogenous bargaining power β ,*

(a) *as long as m_b is sufficiently large, the optimal hiring rule is efficient. That is, for all $t \geq 1$ and all θ^t ,*

$$\begin{cases} h_g(\theta^t) = 1 \\ h_b(\theta^t) = 0 \end{cases}.$$

(b) *for any $t \geq 1$ and any history of the states of the economy θ^t , we have*

$$w_g(\theta^t, \theta_{t+1}) \geq w_g(\theta^t), \text{ for each } \theta_{t+1} \in \{\theta_H, \theta_L\}.$$

⁶If for some parameter profile (α, θ, β) , there is no employment contract that satisfies both conditions IR_f and IR_w, then we define $\Pi(\alpha, \theta, \beta) = \underline{\Pi}(\beta)$ and $U(\alpha, \theta, \beta) = \underline{U}(\theta, \beta)$.

(c) no wage jumps can take place at bust states, and there is at most one wage jump at the boom state. Moreover, after a boom-state wage jump, the worker's wage becomes $w_H(\beta)$ which solves

$$u(w_H(\beta)) + \delta[k\underline{U}_H(\beta) + (1-k)EU(\beta)] = \underline{U}_H(\beta). \quad (11)$$

Proof. See Appendix. ■

The reasons for the above results are the same as those discussed in Section 3.2. The wage dynamics of the optimal employment contract thus can be characterized by $(w_0^*(\alpha, \beta), w_H(\beta))$, where $w_0^*(\alpha, \beta)$ is the worker's wage in the initial period in which the contract is signed, and $w_H(\beta)$ is determined by Equation (11). If the initial wage is low, that is, $w_0^*(\alpha, \beta) < w_H(\beta)$, then the optimal future wage will remain the initial level $w_0^*(\alpha, \beta)$ until the first boom appears, at which the wage jumps to $w_H(\beta)$ and never changes in the future; if $w_0^*(\alpha, \beta) \geq w_H(\beta)$, then the optimal wage is equal to $w_0^*(\alpha, \beta)$ for all periods as long as the worker is retained, regardless of the history of the states of the economy. Hence, to capture the optimal employment contract, it suffices to characterize the optimal initial wage $w_0^*(\alpha, \beta)$.

For notational simplicity, denote

$$\begin{aligned} \underline{U}_L(\beta) &= \underline{U}(\theta_L, \beta); \\ \underline{U}_H(\beta) &= \underline{U}(\theta_H, \beta); \\ EU(\beta) &= p\underline{U}_H(\beta) + (1-p)\underline{U}_L(\beta); \\ \bar{\theta} &= p\theta_H + (1-p)\theta_L. \end{aligned}$$

We define $w_0(\beta)$, $\underline{w}_{0L}(\beta)$, $\underline{w}_{0H}(\alpha, \beta)$, $\bar{w}_{0L}(\alpha, \beta)$ and $\bar{w}_{0H}(\alpha, \beta)$ according to the following conditions.

$$u'(w_0(\beta)) = \frac{\beta}{1-\beta}; \quad (12)$$

$$u(\underline{w}_{0L}(\beta)) + \delta EU(\beta) = \underline{U}_L(\beta); \quad (13)$$

$$u(\underline{w}_{0H}(\alpha, \beta)) + \frac{\alpha\delta[ku(\underline{w}_{0H}(\alpha, \beta)) + (1-k)EU(\beta)]}{1-\delta k} + (1-\alpha)\delta EU(\beta) = \underline{U}_H(\beta); \quad (14)$$

$$\bar{w}_{0L}(\alpha, \beta) = \begin{cases} \frac{\alpha[\frac{m_g + \delta k \bar{\theta}}{1-\delta k} + m_b - \frac{\delta k p w_H(\beta)}{(1-\delta k)(1-\delta k(1-p))}] - (m_b - \theta_L) - \underline{\Pi}(\beta)}{1 + \frac{\alpha \delta k(1-p)}{1-\delta k(1-p)}} & \text{if } \alpha < \hat{\alpha}_L(\beta) \\ \frac{\alpha(\frac{m_g + \delta k \bar{\theta}}{1-\delta k} + m_b) - (m_b - \theta_L) - \underline{\Pi}(\beta)}{1 + \frac{\alpha \delta k}{1-\delta k}} & \text{if } \alpha \geq \hat{\alpha}_L(\beta) \end{cases}; \quad (15)$$

$$\bar{w}_{0H}(\alpha, \beta) = \frac{\alpha(\frac{m_g + \delta k \bar{\theta}}{1-\delta k} + m_b) - (m_b - \theta_H) - \underline{\Pi}(\beta)}{1 + \frac{\alpha \delta k}{1-\delta k}}. \quad (16)$$

With the above definitions, the optimal initial wage $w_0^*(\alpha, \beta)$ can be characterized by the following Proposition.

Proposition 4 *For any $\beta \in [0, 1]$, the optimal initial wage is characterized as follows.*

(a) *Starting with a boom state, let $w_{0H}^*(\alpha, \beta)$ be the optimal initial wage, then*

$$w_{0H}^*(\alpha, \beta) = \begin{cases} \underline{w}_{0H}(\alpha, \beta) & \text{if } w_0(\beta) \leq \underline{w}_{0H}(\alpha, \beta) \leq \bar{w}_{0H}(\alpha, \beta) \\ w_0(\beta) & \text{if } \underline{w}_{0H}(\alpha, \beta) \leq w_0(\beta) \leq \bar{w}_{0H}(\alpha, \beta) \\ \bar{w}_{0H}(\alpha, \beta) & \text{if } \underline{w}_{0H}(\alpha, \beta) \leq \bar{w}_{0H}(\alpha, \beta) \leq w_0(\beta) \end{cases} .$$

Moreover, the match quality $\alpha_H(\beta)$ of the marginal worker hired in a boom state is given by

$$\underline{w}_{0H}(\alpha, \beta) = \bar{w}_{0H}(\alpha, \beta). \quad (17)$$

(b) *Starting with a bust state, let $w_{0L}^*(\alpha, \beta)$ be the optimal initial wage, then*

$$w_{0L}^*(\alpha, \beta) = \begin{cases} \underline{w}_{0L}(\beta) & \text{if } w_0(\beta) \leq \underline{w}_{0L}(\beta) \leq \bar{w}_{0L}(\alpha, \beta) \\ w_0(\beta) & \text{if } \underline{w}_{0L}(\beta) \leq w_0(\beta) \leq \bar{w}_{0L}(\alpha, \beta) \\ \bar{w}_{0L}(\alpha, \beta) & \text{if } \underline{w}_{0L}(\beta) \leq \bar{w}_{0L}(\alpha, \beta) \leq w_0(\beta) \end{cases} .$$

Moreover, the match quality $\alpha_L(\beta)$ of the marginal worker hired in a boom state is given by

$$\underline{w}_{0L}(\beta) = \bar{w}_{0L}(\alpha, \beta). \quad (18)$$

Proof. See Appendix. ■

Equation (12) states that $w_0(\beta)$ is the unconditional optimal initial wage when both (IR_f) and (IR_w) are slack. The concavity of the utility function $u(\cdot)$ then implies that as long as both (IR_f) and (IR_w) are satisfied, the optimal initial wage should be as close to $w_0(\beta)$ as possible.

By Equation (13) and (14), $\underline{w}_{0L}(\beta)$ is the initial wage level at a bust state that generates the outside option $\underline{U}_L(\beta)$, given the wage dynamics in the future; $\underline{w}_{0H}(\alpha, \beta)$ is the fixed wage level that generates the outside option $\underline{U}_H(\beta)$, given that the initial state is boom and the signal is α . Thus, $\underline{w}_{0L}(\beta)$ and $\underline{w}_{0H}(\alpha, \beta)$ can be interpreted as the minimum initial wage levels for (IR_w) to be satisfied, when the initial state is bust and boom, respectively. Similarly, according to definitions (15) and (16), given the optimal wage dynamics, $\bar{w}_{0L}(\alpha, \beta)$ and $\bar{w}_{0H}(\alpha, \beta)$ are the initial wage levels such that the firm earns exactly its outside option, when the initial state is bust and boom, respectively. Thus, $\bar{w}_{0L}(\alpha, \beta)$ and $\bar{w}_{0H}(\alpha, \beta)$ can be interpreted as the maximum initial wage levels for (IR_f) to be satisfied.

Note that $\underline{w}_{0L}(\beta)$ does not depend on α . One can easily check that $\underline{w}_{0H}(\alpha, \beta)$ is decreasing in α , with $\underline{w}_{0H}(\alpha = 1, \beta) = w_H(\beta)$; $\bar{w}_{0L}(\alpha, \beta)$ and $\bar{w}_{0H}(\alpha, \beta)$ are increasing in α , with $\bar{w}_{0L}(\hat{\alpha}_L, \beta) = w_H(\beta)$ determining the threshold value $\hat{\alpha}_L(\beta)$. This means that when $\alpha < \hat{\alpha}_L(\beta)$, we have $\bar{w}_{0L}(\alpha, \beta) < w_H(\beta)$ so that there will be one wage jump at the first boom state; and when $\alpha \geq \hat{\alpha}_L(\beta)$, we have $\bar{w}_{0L}(\alpha, \beta) \geq w_H(\beta)$ so that there is no wage jump and the wage level is $\bar{w}_{0L}(\alpha, \beta)$ for all periods. These properties guarantee the uniqueness of the solutions to equations (17) and (18).

According to Proposition 4, the exogenous bargaining power β determines the optimal initial wage levels. When the optimal initial wage equals the minimum wage level, the worker's initial IR constraint becomes binding, which means that the optimal employment contract is set as if the firm had the full bargaining power. On the other hand, when the optimal initial wage equals the maximum wage level, the firm's initial IR constraint is binding, so that we are equivalently considering a situation where the worker has the full bargaining power. Situations in which both the firm and the worker have some bargaining power can occur only when the optimal initial wage is equal to $w_0(\beta)$. The following Corollary states that our main results still holds as long as the exogenous bargaining power β is larger than some threshold.

Corollary 1 *For any β that satisfies*

$$\frac{\beta}{1 - \beta} \geq u'(\theta_L), \quad (19)$$

the optimal employment contract is exactly the same as that when $\beta = 1$. Thus, the marginal worker hired in a boom state has a lower expected match quality than the marginal worker hired in a bust state:

$$\alpha_H(\beta) < \alpha_L(\beta),$$

so that the cohort effect holds.

Proof. See Appendix. ■

Corollary 1 shows that the validity of these results do not simply rely on the assumption that firms have the full bargaining power. When the exogenous bargaining power β is large, the firm's expected profit becomes so important that the optimal employment contract only leaves the worker with his outside option. Specifically, if (IR_w) is not binding, reducing the initial wage would be strictly profitable for the social planner. This is true no matter the initial state is boom or bust. Hence, as long as condition (19) is satisfied, the social planner only have to maximize the firm's expected profit, acting as if all the bargaining power was on the firm's side.

To compare the marginal hires' match qualities, $\alpha_H(\beta)$ and $\alpha_L(\beta)$, consider a worker who is a marginal bust hire with a match quality $\alpha_L(\beta)$. The bust-state optimal employment contract gives

the worker just his outside option $\underline{U}_L(\beta)$.⁷ For the same pair of the worker and the firm, consider a change of the initial state from θ_L to θ_H . The marginal boom hire has a lower match quality, $\alpha_H(\beta) < \alpha_L(\beta)$, if and only if the optimal contract after the change still satisfies both constraints (IR_f) and (IR_w). Equivalently, we only need to check if there is a feasible contract such that constraint (IR_f) is binding and constraint (IR_w) is satisfied. For (IR_w) to be still satisfied after the change, the feasible contract should give the worker an expected payoff no less than $\underline{U}_H(\beta)$. In other words, the worker's expected payoff in the initial period should be increased by at least $\underline{U}_H(\beta) - \underline{U}_L(\beta) = u(\theta_H) - u(\theta_L)$.⁸ On the other hand, the change of the initial state increases the total output by $\theta_H - \theta_L$, which is the total amount that can be added to the worker's wages in all periods. So the question becomes, whether the best allocation of $\theta_H - \theta_L$ to worker's inter-period wages increases his total utility by at least $u(\theta_H) - u(\theta_L)$?⁹

When β satisfies condition (19), the optimal initial wage of the marginal bust hire satisfies $\underline{w}_{0L}(\beta) = \theta_L$.¹⁰ In this case, the answer to the above question is yes. In fact, such a feasible contract can be obtained by just increasing the initial wage from θ_L to θ_H . This shows that the optimal contract after the change satisfies both (IR_f) and (IR_w), which proves that $\alpha_H(\beta) < \alpha_L(\beta)$.

However, if condition (19) is not satisfied, the marginal bust hire's optimal initial wage $\underline{w}_{0L}(\beta)$ becomes strictly higher than θ_L . One cannot find a feasible contract by simply adding all the increased output $\theta_H - \theta_L$ to the worker's initial wage, just as before. This is because due to the strict concavity of the utility function $u(\cdot)$, the increased utility, $u(\underline{w}_{0L}(\beta) + \theta_H - \theta_L) - u(\underline{w}_{0L}(\beta))$, is strictly lower than $u(\theta_H) - u(\theta_L)$, as long as $\underline{w}_{0L}(\beta) > \theta_L$. In this case, the answer to the above question can be either yes or no, depending on both the degree of concavity of $u(\cdot)$ and other parameters.

6 Conclusion

This paper develops a model of turnover and wage dynamics. The main ingredients of the model are insurance, match-specific productivity, and long-term contracting. We characterize the optimal employment contract and show that wages are downward rigid within a firm and termination occurs. We apply the model to study the business cycle's impact on future wages and job mobility. We show that boom hires are on average worse matches and are therefore more likely to leave even if those who remain have higher wages. These predictions shed light on a number of empirical findings about the persistent effects of initial labor market conditions on a worker's subsequent labor market outcomes.

⁷The firm also earns its outside option.

⁸See equations (9) and (10).

⁹Since the worker's utility function is strictly concave, the best allocation is to make the wages as smooth as possible among the periods. So we should first increase the worker's initial wage on top of $\underline{w}_{0L}(\beta)$. If the initial wage level exceeds $w_H(\beta)$ and there is still leftover of $\theta_H - \theta_L$, then we increase all period wages on top of $w_H(\beta)$ until all $\theta_H - \theta_L$ has been distributed.

¹⁰See Proposition 2.

In this paper, we choose a simple setup to illustrate the key mechanism—better state of the economy increases the value of the relationship by enhancing the scope for insurance, resulting in firms being more willing to hire worse matches in booms. The simple setup means that many relevant factors that affect wages and turnover are not included. For example, we did not consider match quality at a vertical level. Vertical mismatches can be more severe in a recession when overqualified workers settle for lower-paying jobs, leading to higher future turnover rates. Understanding the interaction between horizontal and vertical matching will be key in assessing the implications of business cycles on labor markets. It will also be useful for policymakers who help facilitate better matching between firms and workers. More research on this topic is needed.

7 Appendix

In this section, we provide technical proofs for all the lemmas and propositions in the main text.

Proof of Lemma 1. See the proof of part (b) of Lemma 3. Lemma 1 is just a special case with $\beta = 1$. ■

Proof of Proposition 2. For part (a), when the initial state is boom, then there will be no wage jumps in any future states. This is because if otherwise, the worker’s initial-period IR condition would not be satisfied. Thus, in this case, the optimal wage $w_0(\alpha, \theta_H)$ is a fixed level for all periods, as long as the worker is retained. Moreover, since firms have the full bargaining power, the optimal wage $w_0(\alpha, \theta_H)$ should be set so that the worker’s initial IR constraint is binding. The left-hand side of (3) gives the worker’s expected continuation payoff from the initial period. To understand it, note that the first term is the worker’s utility in the initial period. In the next period, with probability $1 - \alpha$, the worker turns out to be a bad match, in which case he will be fired and earn his outside option depending on the state of this period. The expected continuation payoff is thus given by $E\underline{U}$. With probability α , the worker is a good match and retained by the firm. Then using the optimal hiring rule, in any subsequent period t , with probability k^t , the worker is still retained and earns the wage $w_0(\alpha, \theta_H)$; with probability $k^{t-1} - k^t$, the worker leaves the firm exogenously and earn $E\underline{U}$. This proves that the optimal wage $w_0(\alpha, \theta_H)$ is given by equation (3).

For part (b), when the initial state is bust, then the optimal wage dynamics is characterized by $(w_0(\alpha, \theta_L), w_H)$, where $w_0(\alpha, \theta_L)$ is the wage in the initial period, and w_H is the wage level after a boom-state wage jump. Note that if there is wage jump at a boom state, then the worker’s wage will be w_H forever and his expected continuation payoff will be $\underline{U}(\theta_H)$. The left-hand side of (4) gives the worker’s expected continuation payoff after a boom-state wage jump. To see that, the first term is the utility in the current period. In the next period, with probability k , the worker is still retained and the continuation payoff from this period is $\underline{U}(\theta_H)$; with probability $1 - k$, the worker leaves exogenously and the continuation payoff is $E\underline{U}$. Thus, equation (4) determines w_H . Finally, to solve $w_0(\alpha, \theta_L)$, note that since the worker does not have any bargaining power, his initial IR constraint should be binding in the optimal contract. It follows immediately from

(1) that with $w_0(\alpha, \theta_L) = \theta_L$, the worker's expected continuation payoff from the initial period is exactly equal to $\underline{U}(\theta_L)$. This proves that $w_0(\alpha, \theta_L) = \theta_L$. ■

Proof of Lemma 2. To see $\alpha_H < \alpha_L$, note that the firm's payoff is continuous and increasing in α . When the initial state is bust, the optimal contract pays the worker an initial wage θ_L , and this wage jumps to w_H at the first boom state in the future. For a worker with signal α_L , this optimal contract makes the firm just break even. Now for the same pair of the worker and the firm, consider a change of the initial state from bust to boom. In this case, consider such an employment contract: the worker is paid an initial wage θ_H , and this wage jumps to w_H at the first boom state in the future. One can easily check that this contract still makes the firm break even when the initial state is boom. Moreover, all the constraints (IR), (IC_g) and (IC_b) are satisfied. Note that this contract is not optimal (the optimal contract should pay the worker a fixed wage for all periods, as is shown in Proposition 2). This implies that the firm's profit from hiring a worker with signal α_L is strictly positive, and therefore the cutoff α_H must be lower than α_L . ■

Proof of Proposition 3. First, by using (1), (3) and (4), one can easily show that, for any $\alpha \in (0, 1)$,

$$\theta_H > w_0(\alpha, \theta_H) > w_H > \theta_L, \quad (20)$$

with

$$\begin{aligned} w_0(\alpha = 0, \theta_H) &= \theta_H; \\ w_0(\alpha = 1, \theta_H) &= w_H. \end{aligned}$$

For part (a), according to Proposition 2, we have that, for $t \geq 1$,

$$E_{\alpha, \theta^t}[w_g(\theta^t)|\alpha \geq \alpha_H, \theta_0 = \theta_H] = E_{\alpha}[w_0(\alpha, \theta_H)|\alpha \geq \alpha_H],$$

and

$$E_{\alpha, \theta^t}[w_g(\theta^t)|\alpha \geq \alpha_L, \theta_0 = \theta_L] = pw_H + (1 - p)\theta_L.$$

Thus $E_{\alpha, \theta^t}[w_g(\theta^t)|\alpha \geq \alpha_H, \theta_0 = \theta_H] > E_{\alpha, \theta^t}[w_g(\theta^t)|\alpha \geq \alpha_L, \theta_0 = \theta_L]$ follows immediately from (20).

To prove the second inequality, note that $w_b(\theta^t) = \theta_t$, for any $t \geq 1$ and θ^t . Then we have

$$\begin{aligned} E_{\alpha, \theta^t}[w_b(\theta^t)|\alpha \geq \alpha_H, \theta_0 = \theta_H] & \\ &= E_{\alpha, \theta^t}[w_b(\theta^t)|\alpha \geq \alpha_L, \theta_0 = \theta_L] \\ &= p\theta_H + (1 - p)\theta_L, \end{aligned}$$

which finishes the proofs for part (a).

For part (b), note that according to (20), regardless of the initial states, voluntary(involuntary) turnover is equivalent to turnover that happens at boom(bust) states. Given that the worker has

signal α , the probability that turnover occurs in the next period is $1 - \alpha k$, so that the probability of voluntary turnover is $p(1 - \alpha k)$; and the probability of involuntary turnover is $(1 - p)(1 - \alpha k)$. That is,

$$\begin{aligned} \Pr[\text{turnover with } w_b(\theta_0, \theta_1) > w_0(\alpha, \theta_H) | \alpha \geq \alpha_H, \theta_0 = \theta_H] &= E_\alpha [p(1 - \alpha k) | \alpha \geq \alpha_H]; \\ \Pr[\text{turnover with } w_b(\theta_0, \theta_1) > w_0(\alpha, \theta_L) | \alpha \geq \alpha_L, \theta_0 = \theta_L] &= E_\alpha [p(1 - \alpha k) | \alpha \geq \alpha_L]; \\ \Pr[\text{turnover with } w_b(\theta_0, \theta_1) \leq w_0(\alpha, \theta_H) | \alpha \geq \alpha_H, \theta_0 = \theta_H] &= E_\alpha [(1 - p)(1 - \alpha k) | \alpha \geq \alpha_H]; \\ \Pr[\text{turnover with } w_b(\theta_0, \theta_1) \leq w_0(\alpha, \theta_L) | \alpha \geq \alpha_L, \theta_0 = \theta_L] &= E_\alpha [(1 - p)(1 - \alpha k) | \alpha \geq \alpha_L]. \end{aligned}$$

Thus, part (b) can be proved by the facts that $E_\alpha [\alpha | \alpha \geq x]$ is increasing in x and that $\alpha_H < \alpha_L$.

■

Proof of Lemma 3. For part (a), suppose the optimal contract fires a good-match worker in period t . Then the continuation payoffs at the beginning of period t are $\underline{\Pi}(\beta)$ for the firm and $\underline{U}(\theta_t, \beta)$ for the worker, where θ_t is the state of the economy in period t . Now consider a deviation in which (i) the worker is kept in period t and paid a wage θ_t , and (ii) the contract from period $t + 1$ on is specified as if the worker and the firm met in period $t + 1$ with a signal of match quality $\alpha = 1$.

Under this deviation, the worker earns θ_t in the current period, and receives $E_\theta U(\alpha = 1, \theta, \beta)$ in period $t + 1$. So his expected continuation payoff at the beginning of period t becomes $u(\theta_t) + \delta E_\theta U(\alpha = 1, \theta, \beta)$, which is strictly higher than $\underline{U}(\theta_t, \beta)$ according to 9 and 10. Moreover, the firm's expected continuation payoffs at the beginning of period t is $m_g + \delta E_\theta \Pi(\alpha = 1, \theta, \beta) > \underline{\Pi}(\beta)$ according to (8). This implies that this deviation makes both the worker and the firm strictly better off, which proves that a good-match worker should always be kept. When m_b is sufficiently large, the reason for why a bad-match worker should always be fired is exactly the same as that when $\beta = 1$, as discussed in Section 3.1.

For part (b), recall that for any $\theta_0 \in \{\theta_H, \theta_L\}$, $\alpha \in [0, 1]$ and $\beta \in [0, 1]$, the optimal employment contract solves the following constrained program.

$$\max_C \beta \Pi(\alpha, \theta_0, \beta, C) + (1 - \beta) U(\alpha, \theta_0, \beta, C)$$

subject to constraints (IR_f) , (IR_w) , (IC'_g) , (IC'_b) and (7), where $\Pi(\alpha, \theta_0, \beta, C)$ and $U(\alpha, \theta_0, \beta, C)$ are given by (5) and (6), respectively.

To solve the above problem, set the Lagrangian as

$$\begin{aligned}
L = & \beta \Pi(\alpha, \theta_0, \beta, C) + (1 - \beta) U(\alpha, \theta_0, \beta, C) \\
& + \lambda_f [\Pi(\alpha, \theta_0, \beta, C) - \underline{\Pi}(\beta)] + \lambda_w [U(\alpha, \theta_0, \beta, C) - \underline{U}(\theta_0, \beta)] \\
& + \sum_{\{\theta^t\}} \mu_g(\theta^t) \left\{ \begin{aligned} & \sum_{j=t+1}^{\infty} \delta^{j-t} E_{\theta^j | \theta^t} [\sigma_g(\theta^j) u(w_g(\theta^j)) + (\sigma_g(\theta^{j-1}) - \sigma_g(\theta^j)) \underline{U}(\theta_j, \beta)] \\ & + u(w_g(\theta^t)) \sigma_g(\theta^t) - \underline{U}(\theta_t, \beta) \sigma_g(\theta^t) \end{aligned} \right\} \\
& + \sum_{\{\theta^t\}} \mu_b(\theta^t) \left\{ \begin{aligned} & \sum_{j=t+1}^{\infty} \delta^{j-t} E_{\theta^j | \theta^t} [\sigma_b(\theta^j) u(w_b(\theta^j)) + (\sigma_b(\theta^{j-1}) - \sigma_b(\theta^j)) \underline{U}(\theta_j, \beta)] \\ & + u(w_b(\theta^t)) \sigma_b(\theta^t) - \underline{U}(\theta_t, \beta) \sigma_b(\theta^t) \end{aligned} \right\},
\end{aligned}$$

where λ_f and λ_w are the multipliers associated with the constraints (IR_f) and (IR_w) , respectively; and for any history θ^t , $\mu_g(\theta^t)$ and $\mu_b(\theta^t)$ are the multipliers associated with constraints (IC'_g) and (IC'_b) , respectively.

From the Lagrangian, the FOC with respect to w_0 gives that

$$u'(w_0) = \frac{\beta + \lambda_f}{1 - \beta + \lambda_w}. \quad (21)$$

And for any t and θ^t , the FOC with respect to $w_g(\theta^t)$ gives that

$$u'(w_g(\theta^t)) = \frac{\beta + \lambda_f}{1 - \beta + \lambda_w + \sum_{j=1}^t \lambda_g(\theta^j)}, \quad (22)$$

where $\lambda_g(\theta^t) = \frac{\mu_g(\theta^t)}{\alpha \delta^t \text{Pr}[\theta^t] \sigma_g(\theta^t)}$.

Since $\lambda_g(\theta^t) \geq 0$ for any t and θ^t , it follows from (22) and the strict concavity of $u(\cdot)$ that $w_g(\theta^t, \theta_{t+1}) \geq w_g(\theta^t)$, for each $\theta_{t+1} \in \{\theta_H, \theta_L\}$, which completes the proof of part (b). Note that, if $w_g(\theta^t, \theta_{t+1}) > w_g(\theta^t)$, then according to (22) we must have $\lambda_g(\theta^{t+1}) > 0$, which means that constraint (IC'_g) should be binding at history θ^{t+1} . In other words, if there is a wage jump at period $t + 1$, then the worker's continuation payoff from period $t + 1$ should be exactly equal to his outside option.

For part (c), the wage jumps can never take place at bust states because such a wage jump implies that the worker's expected continuation payoff before the jump should be strictly lower than $\underline{U}(\theta_L, \beta)$, which is never possible due to constraint (IR_w) . The analyses in the proof of part (b) also imply that there is at most one wage jump at boom states. And the worker's continuation payoff becomes $\underline{U}(\theta_H, \beta)$ after such a wage jump. This means that $w_H(\beta)$ should be determined by equation (11). ■

Proof of Proposition 4. Let the optimal initial wage be $w_{0S}^*(\alpha, \beta)$, given that the initial state is θ_S , where $S \in \{L, H\}$. According to the Lagrangian defined in the proof of Lemma 3, for any

$S \in \{L, H\}$, we have that

$$\lambda_f > 0 \implies (\text{IR}_f) \text{ is binding} \implies w_{0S}^*(\alpha, \beta) = \bar{w}_{0S}(\alpha, \beta); \quad (23)$$

$$\lambda_w > 0 \implies (\text{IR}_w) \text{ is binding} \implies w_{0S}^*(\alpha, \beta) = \underline{w}_{0S}(\alpha, \beta), \quad (24)$$

which imply that $\lambda_w > 0$ and $\lambda_f > 0$ is never possible. Moreover, the FOC (21) states that

$$u'(w_{0S}^*(\alpha, \beta)) = \frac{\beta + \lambda_f}{1 - \beta + \lambda_w}.$$

Using (23) and (24) and the definition that $u'(w_0(\beta)) = \frac{\beta}{1-\beta}$, we have

$$\lambda_w > 0 \text{ and } \lambda_f = 0 \implies w_0(\beta) < w_{0S}^*(\alpha, \beta) = \underline{w}_{0S}(\alpha, \beta) \leq \bar{w}_{0S}(\alpha, \beta); \quad (25)$$

$$\lambda_w = 0 \text{ and } \lambda_f > 0 \implies \underline{w}_{0S}(\alpha, \beta) \leq \bar{w}_{0S}(\alpha, \beta) = w_{0S}^*(\alpha, \beta) < w_0(\beta); \quad (26)$$

$$\lambda_w = 0 \text{ and } \lambda_f = 0 \implies w_{0S}^*(\alpha, \beta) = w_0(\beta) \in [\underline{w}_{0S}(\alpha, \beta), \bar{w}_{0S}(\alpha, \beta)]. \quad (27)$$

Since both λ_f and λ_w are nonnegative, one of the above three cases must be true. Thus, claims (25), (26) and (27) imply that

$$\begin{aligned} w_0(\beta) &\leq \underline{w}_{0S}(\alpha, \beta) \leq \bar{w}_{0S}(\alpha, \beta) \implies \lambda_w > 0 \text{ and } \lambda_f = 0; \\ \underline{w}_{0S}(\alpha, \beta) &\leq w_0(\beta) \leq \bar{w}_{0S}(\alpha, \beta) \implies \lambda_w = 0 \text{ and } \lambda_f = 0; \\ \underline{w}_{0S}(\alpha, \beta) &\leq \bar{w}_{0S}(\alpha, \beta) \leq w_0(\beta) \implies \lambda_w = 0 \text{ and } \lambda_f > 0, \end{aligned}$$

which finishes the proof regarding the optimal initial wage.

Finally, (17) and (18) follow immediately from the fact that $\bar{w}_{0S}(\alpha, \beta) - \underline{w}_{0S}(\alpha, \beta)$ is increasing in α , for each $S \in \{L, H\}$. ■

Proof of Corollary 1. We only have to prove that condition (19) guarantees that the constraint (IR_w) is binding regardless of the initial states, so that the labor market behaves as if firms had the full bargaining power.

It follows from definitions (13) and (14) that $\underline{w}_{0L}(\beta) < \underline{w}_{0H}(\alpha, \beta)$. Then according to Proposition 4, we need to show that

$$\frac{\beta}{1-\beta} \geq u'(\theta_L) \implies w_0(\beta) \leq \underline{w}_{0L}(\beta).$$

To prove this, by definition (12), we have $\frac{\beta}{1-\beta} \geq u'(\theta_L) \implies w_0(\beta) \leq \theta_L$. On the other hand, equations (10) and (13) imply that $\underline{w}_{0L}(\beta) \geq \theta_L$. This proves that $w_0(\beta) \leq \underline{w}_{0L}(\beta)$. ■

References

- [1] BAKER, GEORGE, MICHAEL GIBBS, AND BENGT HOLMSTROM. 1994. The Wage Policy of a Firm, *Quarterly Journal of Economics*, 109(4): 921-955.
- [2] BEAUDRY, PAUL AND JOHN DINARDO. 1991. The Effect of Implicit Contracts on the Movement of Wages over the Business Cycle: Evidence from Microdata, *Journal of Political Economy*, 99(4): 665-688.
- [3] BOWLUS, AUDRA. 1995. Matching Workers and Jobs: Cyclical Fluctuations in Match Quality, *Journal of Labor Economics*, 13(2): 335-350.
- [4] BRUNNER, BEATRICE AND ANDREAS KUHN. 2010. The Impact of Labor Market Entry Conditions on Initial Job Assignment, Human Capital Accumulations, and Wages, *Working Paper*.
- [5] FARBER, HENRY. 1994. The Analysis of Inter-Firm Mobility, *Journal of Labor Economics*, 12(4): 554-593.
- [6] GIBBONS, ROBERT AND MICHAEL WALDMAN. 2004. Task-specific Human Capital, *American Economic Review*, 94(2): 203-207.
- [7] GIBBONS, ROBERT AND MICHAEL WALDMAN. 2006. Enriching a Theory of Wage and Promotion Dynamics Inside Firms, *Journal of Labor Economics*, 24(1): 59-107.
- [8] GIBBS, MICHAEL AND WALLACE HENDRICKS. 2004. Do Formal Salary Systems Really Matter? *Industrial and Labor Relations Review*, 58(1): 71-93.
- [9] KAHN, LISA. 2008. Job Durations, Match Quality and the Business Cycle: What We can Learn from Firm Fixed Effects? *Working Paper*.
- [10] KAHN, LISA. 2010. The Long-Term Labor Market Consequences of Graduating from College in a Bad Economy, *Labour Economics*, 17(2): 303-316.
- [11] HALL, ROBERT. 2005. Job Loss, Job Finding, and Unemployment in the U.S. Economy over the Past Fifty Year, in Mark Gertler and Kenneth Rogoff, eds., *NBER Macroeconomics Annual 2005*: 101-137.
- [12] HARRIS, MILTON AND BENGT HOLMSTROM. 1982. A Theory of Wage Dynamics, *Review of Economic Studies*, 49(3): 315-333.
- [13] JOVANOVIC, BOYAN. 1979. Job Matching and the Theory of Turnover, *Journal of Political Economy*, 87(5): 972-990.
- [14] MCLAUGHLIN, KENNETH AND MARK BILS. 2001. Inter-Industry Mobility and the Cyclical Upgrading of Labor, *Journal of Labor Economics*, 19(1): 94-135.

- [15] OKUN, ARTHUR. 1973. Upward Mobility in a High-Pressure Economy, *Brookings Papers of Economic Activity*, 1: 207-252.
- [16] OREOPOULOS, PHILIP, TILL VON WACHTER, AND ANDREW HEISZ. 2012. The Short- and Long-Term Career Effects of Graduating in a Recession, *American Economic Journal: Applied Economics*, 4(1): 1-29.
- [17] OYER, PAUL. 2006. Initial Labor Market Conditions and Long-Term Outcomes for Economists, *Journal of Economic Perspectives*, 20(3): 143-160.
- [18] OYER, PAUL. 2008. The Making of an Investment Banker: Macroeconomic Shocks, Career Choice, and Lifetime Income, *Journal of Finance*, 63(6): 2601-2628.
- [19] REDER, MELVIN. 1955. The Theory of Occupational Wage Differentials, *American Economic Review*, 45(5): 833-852.
- [20] SCHMIEDER, JOHANNES AND TILL VON WACHTER 2010. Does Wage Persistence Matter for Employment Fluctuations? Evidence from Displaced Workers, *American Economic Journal: Applied Economics*, 2(3): 1-21.