When Does Aftermarket Monopolization Soften Foremarket Competition?*

Yuk-fai Fong[†], Jin Li[‡], and Ke Liu[§]

December 4, 2015

Abstract

This paper investigates firms' abilities to tacitly collude when they each monopolize a proprietary aftermarket. When firms' aftermarkets are completely isolated from foremarket competition, they cannot tacitly collude more easily than single-product firms. However, when their aftermarket power is contested by foremarket competition as equipment owners view new equipment as a substitute for their incumbent firm's aftermarket product, profitable tacit collusion is sustainable among a larger number of firms. Conditions under which introduction of aftermarket competition hinders firms' ability to tacitly collude are characterized.

Key Words: Constrained Aftermarket Power, Tacit Collusion, Aftermarket Competition

JEL Codes: L12, L13, L41

1 Introduction

In 1992, the U.S. Supreme Court ruled in favor of eighteen independent service organizations (ISOs), which sued *Kodak* for its refusal to sell them replacement parts for servicing *Kodak*'s photocopiers and micrographics equipment.[1] Since *Kodak*'s market share was not considered substantial in the case, the questions of particular interest to economists are: (i) whether firms that do not have substantial market power in the equipment market are able to exercise their power in the related proprietary aftermarkets; and (ii) whether these firms can earn substantial overall industry profits and cause significant consumer injury.

Borenstein, Mackie-Mason, and Netz (1995, 2000) show that equipment manufacturers tend to set supranormal prices in their proprietary aftermarkets. On the other hand, Shapiro and Teece (1994) and Shapiro (1995) argue that installed-base opportunism is unlikely if equipment manufacturers have reputation concern. While these studies provide different answers to Question (i), they largely agree on Question (ii) in that as long as the equipment market is competitive, firms with monopolized proprietary aftermarkets cannot earn supranormal profits because competition in the equipment market would induce them to rebate any aftermarket profits through lower equipment prices.

To demonstrate the relevance of proprietary aftermarkets to firms' overall profits and consumer welfare, this paper investigates the effect of aftermarket power on equipment sellers' ability to collude. We explicitly distinguish between two types of aftermarket power: *unconstrained aftermarket power* and *constrained aftermarket power*. A firm's aftermarket power is said to be unconstrained if the firm's aftermarket product is for *adding new functionality* that cannot be provided by the equipment. Therefore, a firm's established customers do not consider new equipment as a substitute for the firm's aftermarket product and firms' aftermarkets are isolated from competition in the equipment market. For example, hotel owners possess unconstrained aftermarket power over room service and mini-bar items because guests in need of a late-night refreshment for practical purposes do not consider renting another hotel room as a substitute. Other products characterized by unconstrained aftermarket power include insurance for car rental, memory and hard drive upgrades for laptops, and business class upgrades for air travel. On the other hand, a firm's aftermarket power is said to be constrained if the firm's aftermarket product is for *restoring the lost functionality* of the equipment and its established customers

---

[1] Details of the case are available, for example, in Hay (1993). Borenstein et al. (2000) point out that at the time their paper was published, there were over twenty antitrust cases brought against equipment manufacturers whose customers relied on them heavily for aftermarket supplies/services.

consider new equipment of a different brand as a substitute for the firm's aftermarket product. For example, printer manufactures only enjoy constrained aftermarket power because existing owners of their printer consider its proprietary compatible replacement cartridge and a new printer of another brand as substitutes. Other aftermarket products characterized by constrained aftermarket power include razor blades, air purifier filters, maintenance and repair services for electronic products, luxury watches, vehicles, and expensive medical devices.

This paper shows that when firms' aftermarket power is unconstrained, their ability to sustain supranormal profits is no different from that of single-product firms. Ironically, when their aftermarket power is constrained, collusion is sustainable among a larger number of firms.

We consider oligopolistic firms competing in the equipment market, each the sole provider in the equipment's aftermarket. New customers arrive in the market every period, each staying for two periods. Each customer purchases the equipment in the first period of her market life and the aftermarket product in the second period. Products offered by different firms are *ex ante* homogeneous to consumers in the sense that if consumers anticipate paying the same total price for different firms' equipment and aftermarket products, then consumers in the first period of their market life value these firms' products equally.

When firms enjoy unconstrained aftermarket power, there is no competition between the equipment and aftermarket products. As a result, firms can charge their established customers up to their reservation value for the aftermarket product both in a collusive equilibrium and on the punishment path. If any firm undercuts the equipment price by an infinitesimal amount, it will capture the entire industry profit derived from the life-time consumption of one generation of consumers because following the price cut, the deviating firm will continue to capture the monopoly profit in the aftermarket from the consumers it steals. The fact that a deviating firm is able to *steal the entire industry profit from one generation of customers before losing its share of the profits from all future generations* explains why firms enjoying unconstrained aftermarket power cannot tacitly collude more effectively than single product firms.

When firms enjoy constrained aftermarket power, however, firms' aftermarket power is still subject to competition from the equipment market. In a collusive outcome, firm profits from each generation of customers come from aftermarket sales as well as equipment sales, which take place in different periods. Suppose a deviating firm undercuts the equipment price. By doing so, it is able to capture the entire

3

industry's equipment-sales revenue from the incoming generation of customers. However, it will not be able to capture the entire industry's equilibrium aftermarket sales revenue from the new customers it steals because by the time the deviating firm sells aftermarket services to these customers, the price war in the equipment market has already begun. Since existing equipment owners consider new equipment and aftermarket product as substitutes, the price war in the equipment market will bring down the aftermarket price. It remains true that the deviating firm loses its share of profits from all future generations of customers. The fact that *the deviating firm is unable to capture the entire industry profit from one generation of customers before losing its profits from future generations of customers* explains why tacit collusion is generally easier to sustain among firms possessing constrained aftermarket power than among single-product firms or firms possessing unconstrained aftermarket power.

The price fixing practices of luxury auto makers in China provide a plausible example of behavior consistent with predictions by our theory. In a series of investigations conducted over a three-year period by the China Automobile Dealers Association on behalf of the National Development and Reform Commission, it was found that "several car companies engaged in anti-trust violations including *collusion between manufacturers*, collusion between manufacturers and distributors, and abuse of a dominant market position".[2] These investigations resulted in the Chinese price authorities penalizing FAW-Volkswagen, Chrysler and their respective dealers for concluding and implementing price monopoly agreements. The automakers not only fixed prices of vehicles through their car dealers, but also vertically colluded with their respective dealers to fix the minimum resale price for spare parts to independent mechanics.[3]

We also show that a positive industry profit is sustainable among arbitrarily many firms. While the exact statement of the finding is mostly of theoretical interest, it does suggest that constrained aftermarket power may facilitate collusion even in relatively unconcentrated markets. This result is meaningful because some equipment markets associated with proprietary aftermarket power are indeed unconcentrated, such as the market of luxury watches where there are at least 15 leading brands in which the market leader Rolex's share is less than 19%.[4] Incidentally, several luxury watchmakers were investigated by the EU for colluding to refuse to supply spare parts to independent repairers that did

---

[2] See "Foreign Automakers Face Anti-Trust Scrutiny", english.caixin.com, 08 August, 2014

[3] See "FAW-Volkswagen, Chrysler and Related Dealers Fined Nearly RMB280 Million for Monopolistic Conduct", chinalawvision.com, 11 Nov 2014.

[4] See "Interest from BRICs Fuels World Luxury Watch Market", luxurysociety.com, April 23, 2013.

not belong to the manufacturers' maintenance networks.[5]

Given that constrained aftermarket power facilitates collusion, we also investigate the impact of introducing competition into equipment sellers' aftermarkets. It is shown that aftermarket competition limits the equipment sellers' ability to tacitly collude if the aftermarket product costs significantly less to produce than the equipment, or the industry is relatively unconcentrated. Since the cost of the typical single repair is much lower than the cost of producing the equipment for photocopiers, luxury cars, and luxury watches, our theory can explain why *Kodak* and its competitors, luxury automakers in China, and luxury watchmakers in Europe had the incentive to raise the prices of spare parts or even refuse to sell spare parts to independent repairers.

Shapiro (1995) and Chen, Ross, and Stanbury (1998) offer detailed reviews of earlier aftermarket theories. More recently, Chen and Ross (1999) and Carlton and Waldman (2010) show aftermarket competition can lead to inefficiency. Carlton (2001) and Morita and Waldman (2010) further show that aftermarket monopolization is unlikely to harm or may improve social welfare. By contrast, we demonstrate aftermarket power may facilitate tacit collusion to cause significant consumer injury and provide conditions under which the introduction of aftermarket competition can mitigate consumer injury.

Morita and Waldman (2004) show that, by monopolizing the maintenance market, a durable-goods monopolist can commit to not cutting the product price after having it sold to the customers with the highest willingness to pay. In the current paper, the time-inconsistency problem faced by durable-goods sellers is absent. Also, our focus on collusion among firms is different from theirs.

Ellison (2005) show that firms can collectively benefit from concealing their high add-on prices. Gabaix and Laibson (2006) show that firms' concealment of their add-on prices can be an equilibrium outcome if customers are myopic, i.e., they systematically underestimate their aftermarket consumption and are unaware of their bias, although competition still leads to zero profit. Assuming a weaker notion of consumer myopia,[6] Miao (2010) shows that in a dynamic duopoly model of sellers producing both printers and cartridges, firms will not lower the equipment price enough to dissipate the industry profit. Our aftermarket theory also exploits the competition between printers and cartridges as in Miao (2010),

---

[5] See "EU to investigate alleged collusion by luxury watch-makers", us.fashionmag.com, August 5, 2011.

[6] The myopic consumers in Gabaix and Laibson (2006) are not able to choose whether to consume add-ons after they have purchased the foremarket products, whereas the myopic consumers in Miao (2010) are able to optimize aftermarket consumption.

but differs from these three closely related studies in several significant ways which we further elaborate in Section 5.4.

2 Environment

There are $n \geq 2$ infinitely lived sellers who each produce two products, the *equipment* and the *aftermarket product*, at constant marginal costs C and c, where $0 \leq c \leq C$.[7] We use printer, containing an initial cartridge, as a working example of the equipment, and replacement cartridge as a working example of the aftermarket product.

Consumers arrive in overlapping generations. In each period, a continuum of consumers of measure *one* enter the market and each stays in the market for two periods.[8] Firms and consumers have a common discount factor $\delta \in (0,1)$. We call a consumer in the first period of her market life a *new consumer*. If a consumer in the second period of her market life already owns a firm's printer, we call the consumer the firm's *established customer*. Each printer produced by any firm provides a new consumer with a utility of U. A cartridge produced by firm i is compatible only with firm i's printer. An established customer of firm i considers firm i's replacement cartridge and any firm's printer as perfect substitutes, valuing all at U. But any other firm's cartridge has no value to her. So although firms have market power in the cartridge market, they may still face competition in the printer market. For this reason, we call the firms' market power in cartridge markets *constrained aftermarket power*.

We assume that the production of printers and cartridges is socially efficient:

$$C + \delta c < (1 + \delta) U. \tag{E}$$

To ease exposition, we assume that $C < U$. We refer the reader to a previous version of our paper for the case of $C \geq U$. Note that although established customers only value the cartridges of the same brand as their printers, the products produced by all firms are *ex ante homogeneous* to new customers.

---

[7] Our result that constrained aftermarket power facilitates tacit collusion does not crucially depend on the assumption of constant marginal costs. It will be clear that in our setting, constrained aftermarket power facilitates tacit collusion because it lowers the demands for a firm's printers and cartridges when it deviates. This remains true regardless of the shapes of the firms' cost functions.

[8] In Section 2 of the paper's Web Appendix, we generalized the model to allow consumers to exit the market at a hazard rate strictly between zero and one every period. The appendix is downloadable at http://ihome.ust.hk/~yfong/aftermarket_appendix.pdf.

In each period $t \in \mathbb{N}$, each firm $i \in \{1, 2, ..., n\}$ simultaneously announces the printer price $P_{i,t}$ and, if it has established customers, the cartridge price $p_{i,t}$.[9] In each period, after the prices are announced, new consumers and established customers make their purchase decisions. We assume firms maximize the discounted value of their profits and consumers maximize the discounted value of their payoffs. We also assume that consumers are sophisticated enough to fully understand firms' pricing strategies.

Throughout the paper, we restrict our attention to symmetric, stationary subgame perfect Nash equilibria. More specifically, we look at collusive equilibria in which firms set identical printer-cartridge price vectors on the equilibrium path and any deviation from the *on-the-equilibrium-path prices* triggers a punishment path on which firms perpetually charge the *punishment-path prices*. For this reason, the time and firm subscripts for prices are dropped for ease of exposition.

3   Unconstrained Aftermarket Power

To build a benchmark for comparison, in this section, we modify the main model by assuming that a consumer derives utility from the equipment only in the first period of her market life. In the second period of her market life, if the consumer already owns the equipment, she can derive utility from the aftermarket product provided by her equipment manufacturer but not from any new equipment. We say that equipment sellers enjoy *unconstrained aftermarket power* and we call the aftermarket products *add-ons* in this case. Let the utility from the add-on be $U > c$. Without facing competition from the equipment market, firms can charge their established customers up to $p = U$ regardless of the condition in the equipment market.

Competitive Equilibrium and Punishment Path: In a zero-profit equilibrium, firms necessarily charge $p^C = U$ for the add-on; otherwise a firm can generate a positive profit for at least one period by raising its add-on price. The zero-profit condition $\left(P^C - C\right) + \delta\left(p^C - c\right) = 0$ further implies that $P^C = C - \delta\left(U - c\right)$. We assume that firms revert to the zero-profit equilibrium perpetually following any deviation from a collusive outcome.

Collusive Prices and Profit: Now suppose firms tacitly collude on a price pair (P, p). Expecting to pay $p \leq U$ for the add-on in the following period, new consumers are willing to pay up to $U + \delta\left(U - p\right)$

---

[9]In Section 5.2 we show that constrained aftermarket power continues to facilitate tacit collusion even when firms can commit to a cartridge price. We also investigate the intermediate case in which firms can make a commitment to a cartridge price but have to do so inefficiently in Section 3 of the paper's Web Appendix, which is downloadable at http://ihome.ust.hk/~yfong/aftermarket_appendix.pdf.

for the equipment. For firms to earn positive profits from a customer's life-cycle demands, it is required that $P - C + \delta\left(p - c\right) > 0$. For any $p \leq U$ and $P \in \left(C - \delta\left(p - c\right), U + \delta\left(U - p\right)\right]$, the discounted value of a firm's share of the industry profit is $\frac{P - C + \delta(p - c)}{n(1 - \delta)}$.

Deviation Profit: If a firm deviates, in all future periods, *all* firms that have established customers will charge U for the add-on. Anticipating that, new consumers will not accept any deviating offer with $P' > U$, but a deviation price $P' < \min\left\{U, P\right\}$ will attract all incoming customers. So the deviation profit from new consumers is $\left(\min\left\{U, P\right\} - C\right) + \delta\left(U - c\right)$. Moreover, the deviating firm will also immediately raise its price to its $1/n$ established customers from p to U to earn an extra amount of $\left(U - p\right)/n$. It will then earn no profit from any future generation. Therefore, it is incentive compatible for firms to charge the equilibrium prices if and only if[10]

$$\frac{P - C + \delta\left(p - c\right)}{n\left(1 - \delta\right)} \geq \left(\min\left\{U, P\right\} - C\right) + \delta\left(U - c\right) + \frac{U - p}{n}. \tag{1}$$

Sustainability of Collusion: By raising p and lowering P while keeping $\pi = P - C + \delta\left(p - c\right)$ constant, firms can lower both the deviation profits from the new consumers and from the established customers. Therefore, to most effectively sustain any profit level, firms will set $p = U$. Given $p = U$, consumers are willing to pay up to U for the equipment. Plugging $p = U$ and $P \leq U$ back into (1), we can see that tacit collusion for any positive profit is sustainable if and only if

$$\frac{P - C + \delta\left(V - c\right)}{n\left(1 - \delta\right)} \geq P - C + \delta\left(V - c\right)$$

$$\Leftrightarrow \qquad n \leq \frac{1}{1 - \delta},$$

which is the same condition under which collusion is sustainable for single-product firms.[11] The key reason that unconstrained aftermarket power does not facilitates collusion is that the onset of a price war in the foremarket does not prevent the deviating firm from selling its add-on to the customers it has stolen at the equilibrium price. In other words, *a deviating firm can capture the entire industry profit from one generation of customers before losing the profits from all future generations of customers*, just as in the case of a single-product market.

---

[10] If we include the deviating firm's equilibrium profit from established customers in the period of deviation, $\left(p - c\right)/n$, the incentive constraint would be equivalently written as

$$\frac{p - c}{n} + \frac{P - C + \delta(p - c)}{n(1 - \delta)} \geq \left(\min\left\{U, P\right\} - C\right) + \frac{V - c}{n} + \delta\left(V - c\right).$$

[11] The appropriate single-product benchmark model is one in which consumers arrive in overlapping generations and have repeat demand for the same product in both periods of their market life.

4 Constrained Aftermarket Power

Now we return to the main model in which firms possess constrained aftermarket power. Our main objectives in this section are to (i) show that any positive profit level is sustainable among a larger number of firms when firms possess constrained aftermarket power than when they possess unconstrained aftermarket power or when they sell a single product, and (ii) characterize the range of *steady state per-generation industry profits* that can be supported by tacit collusion for all $\delta \in (0, 1)$ and for all $n \geq 2$. We assume that following any deviation from a collusion outcome, firms revert forever to a SPE play path in which they earn zero profits from each generation of consumers.[12] All proofs are relegated to the Appendix.

4.1 Punishment Path: Zero-Profit Equilibrium

Let (P, p) be an arbitrary printer-cartridge price pair and π be the profit the industry earns from a generation of customers, which we call *per-generation industry profit*. First, in any stationary equilibrium in which firms earn zero profit from each consumer's life-cycle demands, $p = P$ must hold for the following reasons. If $p > P$, then no cartridges would be sold and the per-generation industry profit would be $\pi = P - C$. Zero profit would imply $P = C$. In this case, a firm could earn a positive profit by lowering its cartridge price to some $p' \in (c, C)$ to induce its established customers to purchase the cartridge instead of a new printer. Next, if $p < P$, then the cartridge will be sold in equilibrium and a firm could raise its profit above zero by charging its established customers $p'' \in (p, P)$ for the cartridge.

Furthermore, in any zero-profit equilibrium, all established customers purchase a compatible cartridge. Since $p = P$, if some established customers purchased new printers, then some firm could earn a positive profit by lowering its cartridge price by an infinitesimal amount to induce these established customers to purchase the cartridge instead of the printer. Let p^C denote the common price in a zero-profit

---

[12]The stationary-zero profit equilibrium may not constitute the maximal punishment for the deviating firm. Firms may coordinate on a nonstationary punishment path with the first-period printer price lower than p^C. This will further limit the deviation payoff. If such a punishment path exists and is used by firms, then constrained aftermarket power can facilitate collusion more than our analysis predicts.

equilibrium. Then $\left(p^C - C\right) + \delta\left(p^C - c\right) = 0$, or[13]

$$c < p = P = p^C \equiv \frac{C + \delta c}{1 + \delta} < C. \tag{2}$$

It is easy to verify that no firm has any incentive to deviate.

4.2 Most Effective Collusive Prices and Deviation Profits

The previous subsection described the zero-profit equilibrium. Now we show that constrained aftermarket power helps sustain collusion when the zero-profit equilibrium is used as the off-the-equilibrium-path punishment. Suppose firms collude on a price pair (P, p) and the entire industry's profit from each generation of consumers is $\pi \equiv (P - C) + \delta(p - c) > 0$. By staying on the equilibrium path, each firm earns

$$\frac{p - c}{n} + \frac{\pi}{n\left(1 - \delta\right)} = \frac{(P - C) + (p - c)}{n\left(1 - \delta\right)}.$$

This includes the profit from its established customers and the profit from customers entering the market in the current and all the future periods.

Notice that there are infinitely many combinations of p and P to achieve the same profit level. It is useful to define the *most effective collusive prices* as follows:

Definition 1 Most-Effective Collusive Prices: *For any given per-generation industry profit π, a printer-cartridge price pair (P, p) that yields the per-generation industry profit π is the most effective collusive price pair if and only if it minimizes the deviation payoff.*

It is obvious that if the most effective collusive prices fail to sustain this per-generation industry profit, then there exists no other price pair that can support such a profit, as the alternative prices necessarily lead to a higher deviation payoff. We therefore assume WLOG that firms always adopt the most-effective collusive prices. The following intuitive result simplifies our analysis.

---

[13] Notice that the prediction that the printer and cartridge are sold at the same price is a consequence of the simplifying assumption that they're valued equally by established customers. The key is that the prices are set at a level where established customers are indifferent between purchasing the replacement cartridge and a brand new printer. If established consumers value a new printer at U and a compatible replacement cartridge at V, then in the zero-profit equilibrium, $P = p + U - V$. Nevertheless, it is not uncommon for entry level printers to be priced similar to their replacement cartridges. For example, on May 23, 2007, the Brother HL-2040 Monochrome Laser Printer was sold at Buy.com at $63.99 after a $20 mail-in rebate, while its toner cartridge was priced at $63.98. Of course, competition in the aftermarkets (to be discussed in Section 7) will also alter the relationship between the printer and cartridge prices.

Lemma 1 *The most-effective collusive prices must satisfy $p \leq P$.*

Now consider a firm's deviation payoff. First, look at the case where $p < P$. Since consumers anticipate both the printer and cartridge prices to become $p^C = \frac{C+\delta c}{1+\delta}$ in the period following a unilateral deviation, they will accept a deviation offer P' if and only if $P' < \min\{P, U\}$. The deviating firm can also simultaneously raise the cartridge price up to P' without losing its cartridge business with the measure $1/n$ of established customers. This leads to an instantaneous deviation profit arbitrarily close to

$$(\min\{P, U\} - C) + \frac{\min\{P, U\} - c}{n}.$$

If the firm cuts the printer price further so that P' is arbitrarily close to but less than p, apart from the measure one of new consumers, it also attracts a measure $(n-1)/n$ of competitors' established customers. By doing so, it will earn an instantaneous profit arbitrarily close to $(2n-1)(p-C)/n$. Note that the deviating firm has to lower its cartridge price to P' as well so they won't buy a new printer instead. However, since P' is arbitrarily close to p, the latter price cut does not affect the deviation profit.

Regardless of the price cut, the new consumers the deviator attracts will continue to purchase the cartridge from it at price $p^C = (C + \delta c)/(1 + \delta)$ in the following period, allowing it to earn an additional discounted profit of $\delta(C-c)/(1+\delta)$. Due to the ensuing price war, the deviating firm will not earn any more profit from future generations of customers. This gives rise to the following incentive constraint for firms not to deviate:

$$\frac{p-c}{n} + \frac{\pi}{n(1-\delta)} \geq \max\left\{(\min\{P, U\} - C) + \frac{\min\{P,U\}-c}{n} + \delta\frac{(C-c)}{1+\delta}, \frac{(2n-1)(p-C)}{n} + \frac{p-c}{n} + \delta\frac{(C-c)}{1+\delta}\right\}, \text{ for } p < P. \tag{3}$$

Next, look at the case where firms collude by setting $P = p$. When a deviating firm undercuts the common price for a printer and cartridge by an infinitesimal amount, it attracts the whole generation of new customers as well as all the established customers of its competitors. By cutting the price of its cartridge by the same infinitesimal amount, it can avoid inducing its own established customers to purchase its new printer. Therefore, the incentive constraint for firms not to deviate becomes

$$\frac{p-c}{n} + \frac{\pi}{n(1-\delta)} \geq \frac{(2n-1)(p-C)}{n} + \frac{p-c}{n} + \delta\frac{(C-c)}{1+\delta}, \text{ for } p = P. \tag{4}$$

To collude most effectively, for any given per-generation industry profit level π that the firms target to achieve, firms choose a price pair (P, p) satisfying $(P - C) + \delta(p - c) = \pi$ such that the deviation

11

profit is minimized. This transforms the identification of the most-effective collusive prices into the following problem of minimizing a firm's deviation payoff:

$$\min_{(p,P)} D = \begin{cases} \max\left\{\frac{(n+1)\min\{P,U\}-p-nC}{n} + \delta\frac{(C-c)}{1+\delta}, \frac{(2n-1)(p-C)}{n} + \delta\frac{(C-c)}{1+\delta}\right\} & \text{if } p < P, \\ \frac{(2n-1)(p-C)}{n} + \delta\frac{(C-c)}{1+\delta} & \text{if } p = P, \end{cases} \quad (5)$$

$$\text{subject to } (P-C) + \delta(p-c) = \pi \text{ and } p \leq P.$$

To collude most effectively, for any given π, firms choose a price pair (P,p) satisfying $(P-C) + \delta(p-c) = \pi$ such that the deviation profit is minimized. This is a standard linear programming problem, and the following proposition characterizes the most-effective collusive prices.

Proposition 1 Let $\tilde{\pi} = U + \delta\frac{(n+1)U+(n-1)C}{2n} - C - \delta c$. Then $\delta(C-c) < \tilde{\pi} < \pi^M$, and the most effective prices are

$$(P,p) = \begin{cases} (\pi + \frac{(2n-\delta(n-1))C+2n\delta c-(n+1)\delta U}{2n}, \frac{(n+1)U+(n-1)C}{2n}) & \text{if } \pi \in [\tilde{\pi}, \pi^M], \\ (\frac{2n\pi+(2n-\delta(n-1))C+2n\delta c}{2n+n\delta+\delta}, \frac{(n+1)\pi+2nC+(n+1)\delta c}{2n+n\delta+\delta}) & \text{if } \pi \in [\delta(C-c), \tilde{\pi}), \\ \left(\frac{\pi+C+\delta c}{1+\delta}, \frac{\pi+C+\delta c}{1+\delta}\right) & \text{if } \pi \in [0, \delta(C-c)). \end{cases} \quad (6)$$

Proposition 1 shows that the choice of most effective prices depends on the targeted per-generation industry profit π. When π is high, the most effective collusive price satisfies $P > U > p$, corresponding to Case 1 above. For intermediate π, the most effective collusive price satisfies $U \geq P > p$, corresponding to Case 2. Finally, when π is low, the most effective collusive price satisfies $P = p$, corresponding to Case 3. Notice that when $\pi < \delta(C-c)$, the common price is below the marginal cost of a printer:

$$\frac{\pi + C + \delta c}{1 + \delta} < C \Leftrightarrow \pi < \delta(C-c).$$

This observation is key to our limit result (when the number of firms becomes large) of the characterization of the equilibrium profits in the next section.

4.3 Characterization of the Equilibrium Profit Set

For a given number of firms n and a profit level π, Proposition 1 describes the most effective collusive prices, which allows us to check immediately whether π can be sustained as a SPE. The following theorem builds upon Proposition 1 to characterize the equilibrium profit set:

Theorem 1 There exist $\hat{n}_3 > \hat{n}_2 > \hat{n}_1 > 1/(1-\delta)$, such that the following hold. (i) If $n \leq \hat{n}_1$, then any per-generation industry profit $\pi \in [0, \pi^M]$ can be supported by tacit collusion. (ii) If $n \in (\hat{n}_1, \hat{n}_2]$,

then any per-generation industry profit

$$\pi \in \left[0, \frac{(n-1)(1-\delta)((n+1)\delta+1)\delta(C-c)}{(1+\delta)(2(1-\delta)n^2-(1+2\delta)n-1)}\right] \cup \left[\frac{(1-\delta)(2n-1)(n+1)(U-C)}{2n} + \frac{\delta n(1-\delta)(C-c)}{1+\delta}, \pi^M\right]$$

can be supported by tacit collusion. (iii) *If* $n \in (\hat{n}_2, \hat{n}_3]$, *then any per-generation industry profit*

$$\pi \in \left[0, \frac{(n-1)(1-\delta)((n+1)\delta+1)\delta(C-c)}{(1+\delta)(2(1-\delta)n^2-(1+2\delta)n-1)}\right]$$

can be supported by tacit collusion. (iv) *If* $n > \hat{n}_3$, *then any per-generation industry profit*

$$\pi \in \left[0, \frac{\delta(1-\delta)(n-1)(C-c)}{2(n(1-\delta)-1)}\right]$$

can be supported by tacit collusion.

Figure 1 depicts the set of per-generation industry profits that can be supported by tacit collusion. The curves $\pi = \delta(C-c)$ and $\pi = \tilde{\pi}$ divide the set of feasible profits into three regions as they are divided into three cases in Proposition 1: $\pi \in [0, \delta(C-c))$, $\pi \in [\delta(C-c), \tilde{\pi})$, and $\pi \in [\tilde{\pi}, \pi^M]$. The per-generation profits in different regions are most effectively supported by prices with different expressions reported in Proposition 1.

Figure 1: Set of collusive per-generation industry profits, $C < U$
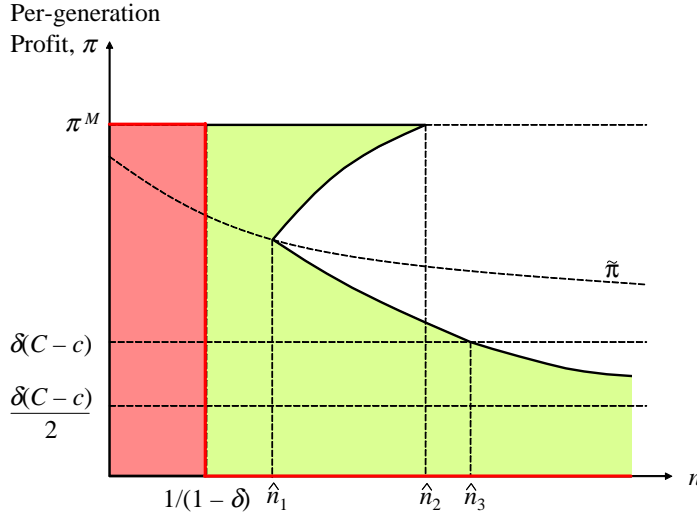
Recall that when firms possess unconstrained aftermarket power, or if they sell a single product, tacit collusion can support up to the monopoly profit whenever $n \leq 1/(1-\delta)$, but firms necessarily earn zero profit otherwise. According to Theorem 1, when firms possess constrained aftermarket power, the full set of feasible profits is sustainable among a larger number of firms, up to $n = \hat{n}_1$ which exceeds

13

$1/(1-\delta)$. Moreover, even when the number of firms exceeds $\hat{n}_1$, the firms can still maintain positive profits.

Now we provide the intuition behind Theorem 1. When each firm possesses constrained aftermarket power, the profit from each generation of customers in part comes from the sale of printers and in part comes from the sale of cartridges which takes place one period later. Consider a positive profit most effectively achieved by setting P and p at some levels satisfying $p^C < p < P$, and that $(P - C)$ is sufficiently larger than $(p - C)$ so that it is optimal for the deviating firm to steal only new consumers instead of both new consumers and competitors' established customers. This corresponds to the case of $\pi \in [\delta(C-c), \tilde{\pi})$ in Proposition 1. By undercutting the printer price, the deviating firm captures the entire industry's printer sales from the incoming new customers. However, when it sells its cartridge to these customers in the following period, the price war will have already begun and the cartridge price will drop from p to p^C. As a result, the deviating firm is unable to capture the entire industry's life-cycle profit from a generation of customers before it loses its equilibrium profits from all future generations of consumers. This comparison makes clear that constrained aftermarket power facilitates tacit collusion. In Fong and Liu (2011), loyalty awards also facilitates collusion by limiting the deviation profit. Apart from the difference in the subjects the two papers study, the deviation profits are also limited differently: loyalty rewards make it immediately costly for firms to steal competitors' repeat customers whereas constrained aftermarket power limits the total deviation profit coming from new consumers through a price war in the equipment market one period after the deviation.

When the industry targets a higher profit level, which corresponds to the case of $\pi \in [\tilde{\pi}, \pi^M)$ in Proposition 1, the profit is most effectively supported by setting the printer price above the consumers' per-period reservation value, i.e., $P > U$. When $P > U$, because consumers are able to anticipate a price war upon seeing a deviation, they will not accept any deviation offer unless the deviating firm discretely drops the price from P to U. Here consumers' rational expectation further facilitates collusion similar to how durable-goods sellers sustain high profits (see, e.g., Ausubel and Deneckere (1987), Gul (1987), and Dutta, Matros, and Weibull (2007)).[14] This explains why when $n \in (\hat{n}_1, \hat{n}_2]$, it is possible to sustain high but not moderate profits.

---

[14]Unlike in these analyses of durable goods sellers, however, *consumer sophistication is unimportant* for our qualitative results. In an earlier version of the paper, we show that even if consumers are unable to anticipate a price war upon seeing a price cut, it remains true that constrained aftermarket power facilitates tacit collusion.

Finally, firms can earn a positive per-generation industry profit by charging the same price for the printer and cartridge, with the common price set below the marginal cost of the printer. This corresponds to the case of $\pi \in [0, \delta(C-c))$ in Proposition 1. Such a loss-leading pricing strategy further weakens the incentive to deviate. When a firm undercuts the common price, it has to incur an immediate loss selling the printer below cost to all the customers it attracts, including competitors' established customers. While the deviating firm can recoup the up-front loss on the printers sold to the new consumers, it would not be able to do so with the printers sold to competitors' established customers because they will leave the market. In fact, for some sufficiently low, yet still profitable, common prices, the deviating firm is unable to recoup the up-front loss regardless of the number of firms. This implies that some positive industry profit can be achieved even when there are arbitrarily many firms. While this is a striking result, we consider it to be mostly of theoretical interest.

Corollary 1 *For all $\delta \in (0,1)$, as n approaches infinity, the set of per-generation industry profit sustainable by tacit collusion converges to $\left[0, \frac{\delta(C-c)}{2}\right]$.*

5 Discussion

5.1 Aftermarket Competition

In the preceding analysis, we showed that constrained aftermarket power facilitates tacit collusion. One natural question to ask is whether taking away firms' aftermarket power can effectively limit their ability to tacitly collude. We investigate this question by introducing competition in firms' aftermarkets. We assume that in a *competitive aftermarket*, p is exogenously fixed at c. This can be interpreted as a regulated price. It can also be endogenized by explicitly introducing independent vendors selling cartridges for each printer. As the number of independent cartridge vendors becomes large enough, in any equilibrium $p = c$.[15]

Proposition A1 in the Appendix pins down precisely when aftermarket competition makes collusion harder to sustain compared to the case of monopolized aftermarkets. This happens when it costs considerably more to produce a printer than a cartridge (i.e., $C > \overline{C}$ for some $\overline{C} \in (c, U)$) *or* when the printer market is relatively unconcentrated (i.e., $C \in (c, \overline{C}]$ and $n > \hat{n}(C)$ for some $\hat{n}(C) \in (2, \infty)$).

---

[15] Miao (2010) endogenizes aftermarket competition by allowing each firm to unilaterally make its printer and cartridge compatible with the competitor's. Here, we take a different approach.

This result may explain why Kodak had a desire to monopolize the aftermarket for its photocopiers. As noted by the district court, Kodak competed with Xerox, IBM, Bell and Howell, 3M and various Japanese manufacturers in the equipment market which some analysts viewed as very competitive (Chen and Ross, 1999). Also, it is reasonable to say that the cost of manufacturing a photocopier was significantly higher than the cost of the typical repair service. Under these conditions, Kodak and its competitors would prefer that the aftermarkets of the printers be monopolized.

On the other hand, the proposition also points out that if at the same time the cost of a printer is sufficiently close to that of a cartridge *and* the printer market is sufficiently concentrated, then aftermarket competition actually promotes collusion.

The intuition of Proposition A1 is as follows. When the aftermarket price is lowered to its marginal cost c, there are two effects on the deviation profit. On the one hand, the deviating firm only has to cut the printer price down to $U + \delta (C - c)$, as opposed to U in the case of a monopolized aftermarket, to attract new customers, which raises the deviation profit. On the other hand, the deviating firm cannot raise the price to its measure $1/n$ of established customers at this moment of deviation, which lowers the deviation profit. When C is high, or when n is large so that the second effect becomes negligible, the first effect dominates. So introducing aftermarket competition makes collusion harder to sustain. However, when C is low and n is small, the second effect dominates, causing the opposite to happen.

5.2   Commitment through Long-Term Contracts

The literature has shown in other contexts that firms' ability to commit to future prices can significantly impact market outcomes (see, e.g., Farrell and Shapiro 1988 and Dana and Fong 2010). In our setting, a firm may use different methods to commit to the aftermarket price. Here we look at the case in which the *commitment is made via a binding long-term contract* which specifies at the time of the printer purchase both the price of the printer, P, and the price of the replacement cartridge (to be delivered in the following period), p.

According to Proposition A2 in the Appendix, allowing for long-term contracts further facilitates the firms' ability to tacitly collude on high profit levels. This is because a deviating firm is unable to raise its cartridge price to its established customers at the time of deviation and it also cannot undercut the equilibrium cartridge price to steal the competitors' established customers who already signed a long-term contract.

When the firms target low profit levels, however, allowing for long-term contracts makes tacit collusion harder to sustain. This happens, in particular, when firms set $P = p$ on the equilibrium path. When firms are allowed to offer long-term contracts, the deviating firm can offer a long-term contract that is attractive to new consumers but not to the competitors' established customers because only new consumers value the future cartridge promised in the long-term contract. For this reason, when firms can deviate by making long-term contract offers, supranormal industry profit is no longer sustainable among arbitrarily many firms.[16]

5.3 Downward Sloping Demands

Under the assumption of unit demands, consumers always consume the same amount of both the equipment and the aftermarket product so the first best is always achieved; any consumer injury caused by aftermarket power is captured by the industry as profit. To discuss the generality of our findings and provide a meaningful welfare analysis, we modify the main model such that consumers have continuous demand for both the (divisible) printer and (divisible) cartridge and downward sloping demand for the cartridge. More specifically, we assume that in the first period of a consumer's market life, she has up to one unit of printing demand and a new consumer's printing demand can only be met by printers. Her willingness to pay for printing is U per unit for up to one unit and has no additional printing demand in the first period. If she purchases less than one unit of printer, she leaves the market in the second period; If she purchases at least one unit of printer from one firm, she is an established customer of that firm. An established customer's willingness to pay for printing is U per unit for the first unit of printing and the demand for additional printing beyond the first unit is captured by the function $v(x)$, with $v(0) = U$ and $v'(x) < 0$ for $x > 0$. Firm i's established customers consider firm i's cartridge and any firm's printer as perfect substitutes, so her printing demand can be met by either firm i's cartridge or any firm's printer.

We argue that in this modified setting, constrained aftermarket power still facilitates tacit collusion. Furthermore, the collusive outcome is more socially efficient than the competitive equilibrium when the discount factor is high but less socially efficient than the competitive equilibrium when the discount factor is moderate or low.

---

[16] In Section 3 of the Web Appendix of the paper, we study the case in which demand is uncertain and quality is not fully verifiable. It is formally shown that under these assumptions, even if firms can deviate by offering a long-term contract, a supranormal profit once again is sustainable among arbitrarily many firms.

A new consumer purchases a full unit of printer from one of the firms if and only if her life-time consumer surplus is positive:

$$U - P + \delta(U - p + \int_0^{v^{-1}(p)} (v(x) - p)\, dx) \geq 0.$$

Although each established consumer's demand for cartridge is downward-sloping, note that consumers are homogeneous in this modified setting.

It can be verified that in the competitive equilibrium firms charge a common price p^C such that

$$\pi = p^C - C + \delta(1 + v^{-1}(p^C))(p^C - c) = 0.$$

In this equilibrium, each firm earns zero profit, but the equilibrium is not socially efficient as $p^C > c$.

We first explain that when firms are sufficiently patient, social efficiency can be achieved and all surplus is captured by the firms. In this collusive equilibrium, firms set $p = c$ and set P at a level that takes away all consumer surplus:

$$U - P + \delta(U - c + \int_0^{v^{-1}(c)} (v(x) - c)\, dx) = 0.$$

Note that while tacit collusion can be efficiency enhancing in this case, consumers are still harmed. Since firms are patient enough, these most profitable prices are sustainable.

Next, we argue that when firms' discount factor is at an intermediate or low level, to effectively tacitly collude, firms set $P \geq p > p^C$ just as in the main model. In this case, since $p > p^C$, tacit collusion actually harms social efficiency. We first explain why it is no longer effective to rely on a high-P-low-p price configuration to capture the collusive profit when δ is not high. On the one hand, a high P allows a deviator to steal high profits from the equipment market. On the other hand, a low p also allows the deviator to raise the cartridge price from p to the smaller of the printer price and the monopoly cartridge price to capture additional profits.

When the discount factor is moderate or low, to sustain otherwise unsustainable collusion, firms move some of the equilibrium profit from the printer market to the cartridge market, and when a firm deviates in the printer market, it is punished in both the printer and cartridge market, just like in the main model. These observations imply that if the firms want to more effectively sustain collusion for moderate or low δ, they would set $p > p^C$, and lead to an outcome less efficient than the competitive equilibrium. Finally, since firms can still move some of the equilibrium profit from the printer market to the cartridge market as in the case of unit demands for printers and cartridges, constrained aftermarket power still facilitates collusion in the modified model.

18

5.4 Empirical Implications and Possible Tests of Theory

A number of empirical implications of our paper allows us to test the validity of the theory and compare it with other existing aftermarket theories. First, our theory predicts the same frequency of collusion and profitability in markets with no aftermarket power and markets with unconstrained aftermarket power and a higher frequency of collusion and higher profits in markets with constrained aftermarket power, after controlling for other factors. To empirically test this implication, one can partition the markets into three categories, respectively, markets with no aftermarket power, markets with unconstrained aftermarket power and markets with constrained aftermarket power,[17] and estimate and compare the frequencies of collusion and firms' profits in the three categories. To identify incidences of collusion, we can adopt an approach employed by Porter and Zona (1993), Porter and Zona (1999) and Bajari and Ye (2003), according to which a high correlation between the unexplained parts of firms' price offers is an indication of collusion. Several other approaches to identify collusion can also be found in Harrington (2008).

The second implication of our theory is that aftermarket competition makes collusion harder to sustain if the difference in costs of the equipment and aftermarket product is relatively large or the equipment market is relatively unconcentrated, and makes collusion easier to sustain if otherwise. This implication can also be tested by estimating the correlation between frequency of equipment seller collusion and the average number of third-party aftermarket suppliers.

Our theory also generates different implications from existing aftermarket theories. According to the add-on pricing literature (e.g. Ellison (2005) and Miao (2010)), firms can earn supranormal profits by shrouding the information of the aftermarket product, while our collusion theory predicts that shrouding prices would make collusion harder to sustain by making deviation harder to detect, and thus lower firm profits. To empirically test which theory fits data better, we can partition markets characterized by constrained aftermarket power into ones that shroud the information of the aftermarket product and ones that do not. Gabaix and Laibson (2006) provide several methods to identify the existence of shrouding and its effects. One method is for researchers to conduct consumer surveys to determine whether consumers are aware of the aftermarket-product cost when making equipment purchases. Another is for researchers to determine whether firms increase the search cost for aftermarket

---

[17]We assume that in many cases it is clear whether aftermarket power is constrained or unconstrained. When it is unclear, it can be determined by estimating the cross-price elasticity of the established customers' demands for the equipment and aftermarket product.

prices. Once firms are categorized using Gabaix and Laibson's methods, we can compare firm profits in these two groups of markets. We can also use controlled field experiments to identify the effect of unshrouding, as also suggested by Gabaix and Laibson.

According to Miao (2010), firm profits are proportional to the switching cost and are zero in the absence of switching cost (Proposition 3), while our model predicts that firms can earn supranormal profits even in the absence of switching costs. If we can classify markets characterized by constrained aftermarket power into high-switching-cost ones and low-switching-cost ones, then we can compare firm profits in these two groups of industries. To measure the magnitude of switching costs, we can use a method employed by Chen and Hitt (2002) and Grzybowski (2008), in which the switching cost is estimated as the effect of switching behavior on consumers' utilities controlling for firm-specific attributes and customer characteristics. Another approach is to estimate a structural model in which switching costs are embedded in firms' pricing decisions (e.g. Kim, Kliger and Vale (2003)).

6 Conclusion

In this paper, we illustrate how equipment sellers can use their constrained aftermarket power to soften competition in the equipment market. The time lag between foremarket and aftermarket consumption and the substitutability between the equipment and aftermarket products prevent a deviating firm from capturing the entire industry profit from a generation of customers before losing the profits from all future generations. Our analysis suggests that it is important to distinguish constrained from unconstrained aftermarket power, as only the former facilitates collusion. Our theory generates a number of empirical implications which allows us to test the validity of the theory and compare it with other existing aftermarket theories.

We believe the competition-softening effect of constrained aftermarket power is very general. First, in the Web Appendix, we show that our main results generalize to the case when consumers do not understand firms' collusive strategies. We also discuss how our findings generalize even if a firm can deviate by offering a long-term contract, if demands for the equipment and the aftermarket product are downward sloping, and if marginal costs are convex instead of constant. We also analyze the impacts of aftermarket competition on equipment sellers' ability to tacitly collude.

Our paper makes the simplifying assumption that there is no brand switching cost. Future research should study whether constrained aftermarket power still facilitates collusion in the presence of brand

switching costs.

Appendix

Proof of Lemma 1. If instead that $p > P$ in equilibrium, then no cartridges would be sold and the per-generation industry profit would be $\pi = (P - C) + \delta (P - C)$. By cutting the printer price below P, a deviating firm could steal all the new and established customers. Suppose firms instead coordinate on the common printer and cartridge price p', where

$$p' = P - \frac{\delta (C - c)}{1 + \delta} < P < p,$$

so that cartridges would be sold in equilibrium and the equilibrium per-generation industry profit would remain at $(P - C) + \delta (P - C)$. However, a deviating firm would be unable to cut the printer price below p' to steal the new and established customers. This would lower the deviation profit. ∎

Proof of Proposition 1. By substituting the rearranged constraint

$$P = \pi + C - \delta (p - c) \tag{7}$$

into the minimization problem (5), the latter can be rewritten as:

$$\min_{p \in [0, \bar{p}]} D(p, \pi) = \begin{cases} \max \{\min \{f_1(p, \pi), f_2(p)\}, f_3(p)\} & \text{if } p < \bar{p}, \\ f_3(p) & \text{if } p = \bar{p}, \end{cases} \tag{8}$$

where $\bar{p} \equiv \frac{\pi + C + \delta c}{1 + \delta}$ and

$$\begin{aligned} f_1(p, \pi) &= \frac{(n+1)(\pi + C - \delta(p-c)) - p - nC}{n} + \frac{\delta(C-c)}{1+\delta}, \\ f_2(p) &= \frac{(n+1)U - p - nC}{n} + \frac{\delta(C-c)}{1+\delta}, \\ f_3(p) &= \frac{(2n-1)(p-C)}{n} + \frac{\delta(C-c)}{1+\delta}. \end{aligned}$$

Notice that

$$\frac{\partial f_1}{\partial p} = -\frac{(n+1)\delta + 1}{n} < \frac{\partial f_2}{\partial p} = -\frac{1}{n} < 0 < \frac{\partial f_3}{\partial p} = \frac{2n-1}{n}. \tag{9}$$

Let $p = \hat{p}_{12}$ solve $f_1(p, \pi) = f_2(p)$, $p = \hat{p}_{13}$ solve $f_1(p, \pi) = f_2(p)$, and $p = \hat{p}_{23}$ solve $f_2(p) = f_3(p)$. It

21

can be verified that

$$\hat{p}_{12} = \frac{\pi - U + C + \delta c}{\delta}, \tag{10}$$

$$\hat{p}_{13} = \frac{(n+1)\pi + 2nC + (n+1)\delta c}{2n + n\delta + \delta}, \tag{11}$$

$$\hat{p}_{23} = \frac{(n+1)U + (n-1)C}{2n}. \tag{12}$$

By applying (9), we can also obtain that

$$
\begin{aligned}
f_1(p, \pi) < f_2(p) && \text{if and only if } p > \hat{p}_{12}, \\
f_1(p, \pi) < f_3(p) && \text{if and only if } p > \hat{p}_{13}, \\
f_2(p) < f_3(p) && \text{if and only if } p > \hat{p}_{23}.
\end{aligned}
\tag{13}
$$

Next, it can be verified that $f_1 = f_2 = f_3$ and $\hat{p}_{12} = \hat{p}_{13} = \hat{p}_{23}$ if and only if

$$\pi = \tilde{\pi} \equiv U + \delta \frac{(n+1)U + (n-1)C}{2n} - C - \delta c.$$

Since

$$\frac{\partial \hat{p}_{23}}{\partial \pi} = 0 < \frac{\partial \hat{p}_{13}}{\partial \pi} = \frac{n+1}{2n + \delta n + \delta} < \frac{\partial \hat{p}_{12}}{\partial \pi} = \frac{1}{\delta},$$

$$
\begin{aligned}
\hat{p}_{12} < \hat{p}_{13} < \hat{p}_{23} && \text{if } \pi < \tilde{\pi}, \\
\hat{p}_{12} \geq \hat{p}_{13} \geq \hat{p}_{23} && \text{if } \pi \geq \tilde{\pi}.
\end{aligned}
\tag{14}
$$

It can be verified that

$$
\begin{aligned}
\pi^M - \tilde{\pi} &= \frac{\delta(n-1)}{2n}(U - C), \\
\tilde{\pi} - \delta(C - c) &= \frac{2n + \delta(n+1)}{2n}(U - C).
\end{aligned}
$$

Since $C < U$ by assumption,

$$\delta(C - c) < \tilde{\pi} < \pi^M \tag{15}$$

(i) First consider the case where $\pi < \tilde{\pi}$ and $\bar{p} < \hat{p}_{13}$. Then according to (14), $\hat{p}_{12} < \hat{p}_{13} < \hat{p}_{23}$. Applying (13), it follows that

$$
\max\{\min\{f_1(p, \pi), f_2(p)\}, f_3(p)\} =
\begin{cases}
f_2(p) & \text{if } p < \hat{p}_{12}, \\
f_1(p, \pi) & \text{if } p \in [\hat{p}_{12}, \hat{p}_{13}), \\
f_3(p) & \text{if } p \geq \hat{p}_{13}.
\end{cases}
$$

According to (9),

22

Lemma A1 *If* $\pi < \tilde{\pi}$*, then* $\max\{\min\{f_1(p,\pi), f_2(p)\}, f_3(p)\}$ *is decreasing in* p *for* $p < \hat{p}_{13}$ *and increasing in* p *for* $p \geq \hat{p}_{13}$.

If $\bar{p} < \hat{p}_{13}$, which holds if and only if

$$\frac{\pi + C + \delta c}{1 + \delta} < \frac{(n+1)\pi + 2nC + (n+1)\delta c}{2n + n\delta + \delta}$$
$$\Leftrightarrow \qquad \pi < \delta(C - c),$$

then

$$
\begin{aligned}
f_3(\bar{p}) &\leq \max\{\min\{f_1(\bar{p},\pi), f_2(\bar{p})\}, f_3(\bar{p})\} \\
&= \min\{f_1(\bar{p},\pi), f_2(\bar{p})\} \\
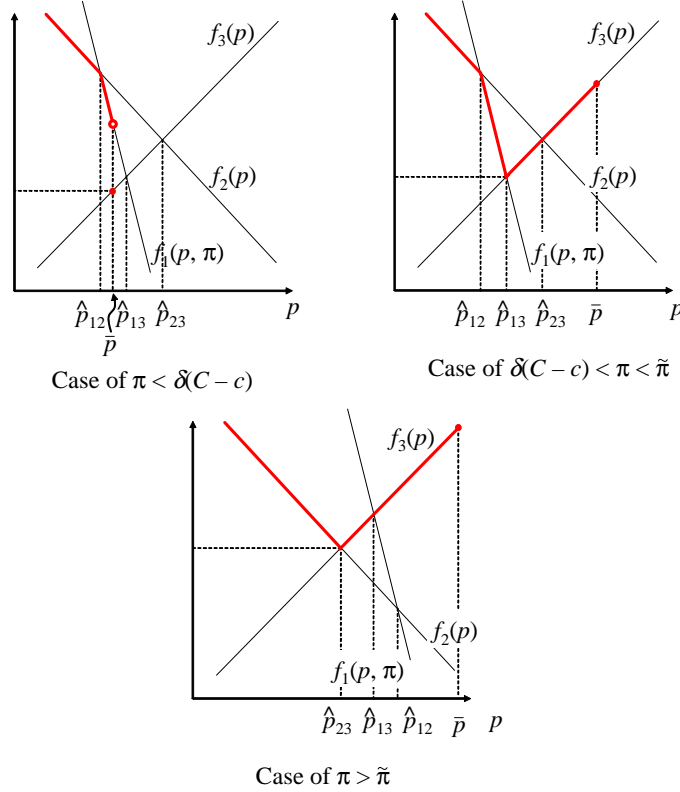&\leq \min\{f_1(p,\pi), f_2(p)\}.
\end{aligned}
$$

The equality is implied by (13) and (14) and the second inequality follows Lemma A1. So, the deviation profit $D(p,\pi)$ is minimized at $p = \bar{p}$.

Figure A1 provides a graphical illustration of the identification of the most effective collusive prices for the case of $C < U$ which covers the sub-cases considered in parts (i)-(iii), although the formal proof

does not utilize the figure. The deviation profit $D(p, \pi)$ is depicted by bolded lines in the figure.

A1

Figure A1: Deviation Profit $D(p, \pi)$, $C < U$

Case of $\pi < \delta(C - c)$

Case of $\delta(C - c) < \pi < \tilde{\pi}$

Case of $\pi > \tilde{\pi}$

2.pdf

(ii) Now look at the case where $\pi < \tilde{\pi}$ and $\hat{p}_{13} \leq \bar{p}$; the latter inequality holds if and only if $\pi \geq \delta(C - c)$. According to Lemma A1, $\max\{\min\{f_1(p, \pi), f_2(p)\}, f_3(p)\}$ is minimized at $p = \hat{p}_{13}$. Since $f_3(\hat{p}_{13}) < f_3(\bar{p})$, as implied by $f_3(\cdot)$ being increasing,

$$\arg\min_{p \in [0, \bar{p}]} D(p, \pi) = \arg\min_{p \in [0, \bar{p}]} \max\{\min\{f_1(p, \pi), f_2(p)\}, f_3(p)\} = \hat{p}_{13}.$$

(iii) Now look at the case where $\pi \geq \tilde{\pi}$. According to (14), $\hat{p}_{12} \geq \hat{p}_{13} \geq \hat{p}_{23}$. Applying (13), it follows that

$$\max\{\min\{f_1(p, \pi), f_2(p)\}, f_3(p)\} = \begin{cases} f_2(p, \pi) & \text{if } p < \hat{p}_{23}, \\ f_3(p) & \text{if } p \in [\hat{p}_{23}, \hat{p}_{12}), \end{cases}$$

which is decreasing in p for $p < \hat{p}_{23}$ and increasing in p for $p \geq \hat{p}_{23}$. Thus, $\max\{\min\{f_1(p, \pi), f_2(p)\}, f_3(p)\}$ is minimized at $p = \hat{p}_{23}$. Since $\pi \geq \tilde{\pi} > \delta(C - c)$, it also follows that $\hat{p}_{23} < \bar{p}$; the latter and the fact that $f_3(\cdot)$ is increasing imply $f_3(\hat{p}_{23}) < f_3(\bar{p})$. Therefore,

$$\arg\min_{p \in [0, \bar{p}]} D(p, \pi) = \arg\min_{p \in [0, \bar{p}]} \max\{\min\{f_1(p, \pi), f_2(p)\}, f_3(p)\} = \hat{p}_{23}.$$

24

Finally, the corresponding printer prices are easily obtained by using (7). This completes the proof of the proposition. ∎

Proof of Theorem 1. From the proof of Proposition 1, we can see that when firms charge the most effective collusive prices, a deviating firm is either forced to undercut the cartridge price (when $p = P$) or indifferent between undercutting the printer price and undercutting the cartridge price (when $p < P$). In other words, given that we assume that firms post the most effective collusive prices, the deviation profit is always

$$f_3(p) = \frac{(2n-1)(p-C)}{n} + \frac{\delta(C-c)}{1+\delta}.$$

This result will be applied repeated in this proof.

The theorem focuses on the case of $U > C$, i.e., $\pi^M > \delta(C-c)$.

Suppose for now the industry targets a per-generation industry profit of $\pi \leq \delta(C-c)$. Applying Proposition 1, the deviation profit is minimized at $P = p = \bar{p} = \frac{\pi+C+\delta c}{1+\delta}$. So, for $\pi \leq \delta(C-c)$, firms' incentive constraint reduces to

$$\frac{\pi}{n(1-\delta)} \geq \frac{(2n-1)}{n}\left(\frac{\pi+C+\delta c}{1+\delta} - C\right) + \delta\frac{C-c}{1+\delta}, \tag{16}$$

which can be rewritten as

$$2(n(1-\delta)-1)\pi \leq \delta(n-1)(1-\delta)(C-c). \tag{17}$$

This incentive constraint is obviously satisfied if $n \leq 1/(1-\delta)$. And for $n > 1/(1-\delta)$, it is easier to satisfy with a lower π. Therefore, the incentive constraint is satisfied for all $\pi \leq \delta(C-c)$ if it is satisfied at $\pi = \delta(C-c)$, i.e.,

$$2(n(1-\delta)-1)\delta(C-c) \leq \delta(n-1)(1-\delta)(C-c)$$
$$\Leftrightarrow \qquad n \leq \frac{1+\delta}{1-\delta} \equiv \hat{n}_3.$$

And for $n > \hat{n}_3$, the set of sustainable profits is characterized by (17). By now we have established the following lemma:

Lemma A2 *For all $n \leq \hat{n}_3$, any profit $\pi \in [0, \delta(C-c)]$ can be supported by tacit collusion. For all $n > \hat{n}_3$, any profit*

$$\pi \in [0, \frac{\delta(n-1)(1-\delta)(C-c)}{2(n(1-\delta)-1)}] \tag{18}$$

can be supported by tacit collusion.

25

Next, suppose the industry targets a per-generation industry profit of $\pi \in [\delta (C - c), \tilde{\pi}]$. According to Proposition 1, the most effective collusive prices are $(P, p) = (\frac{2n\pi + (2n - \delta(n-1))C + 2n\delta c}{2n + n\delta + \delta}, \frac{(n+1)\pi + 2nC + (n+1)\delta c}{2n + n\delta + \delta})$. Therefore, tacit collusion is sustainable if and only if

$$\frac{\pi}{n(1-\delta)} \geq \frac{(2n-1)}{n}\left(\frac{(n+1)\pi + 2nC + (n+1)\delta c}{2n + (n+1)\delta} - C\right) + \delta\frac{(C-c)}{1+\delta}, \tag{19}$$

$$\Leftrightarrow \left(\frac{(2n-1)(n+1)}{2n + (n+1)\delta} - \frac{1}{1-\delta}\right)\pi \leq \frac{(n-1)((n+1)\delta + 1)\delta(C-c)}{(1+\delta)(2n + (n+1)\delta)} \tag{20}$$

The incentive constraint is always satisfied if

$$\frac{(2n-1)(n+1)}{2n + (n+1)\delta} \leq \frac{1}{1-\delta}$$

$$\Leftrightarrow \quad n \leq \frac{(1+2\delta) + \sqrt{4\delta^2 - 4\delta + 9}}{4(1-\delta)}.$$

When n exceeds this critical value, the incentive constrain is easier to satisfy with a lower π. Therefore, it is satisfied for all $\pi \in [\delta(C-c), \tilde{\pi}]$ if it is satisfied at $\pi = \tilde{\pi}$, i.e.,

$$\left(\frac{(2n-1)(n+1)}{2n+(n+1)\delta} - \frac{1}{1-\delta}\right)\left(U + \delta\frac{(n+1)U + (n-1)C}{2n} - C - \delta c\right) \leq \frac{(n-1)((n+1)\delta+1)\delta(C-c)}{(1+\delta)(2n+(n+1)\delta)},$$

which can be rewritten as

$$n \leq \frac{(\delta+1)(U-C+2\delta(U-c)) + \sqrt{(\delta+1)^2(U-C+2\delta(U-c))^2 + 8(1-\delta^2)(U-C+U\delta-c\delta)(U-C)}}{4(1-\delta)(U-C+U\delta-c\delta)} \equiv \hat{n}_1.$$

For $n > \hat{n}_1$, the sustainable profit is bounded from above according to (20):

$$\pi \leq \frac{(n-1)(1-\delta)((n+1)\delta+1)\delta(C-c)}{(1+\delta)(2(1-\delta)n^2 - (1+2\delta)n - 1)}. \tag{21}$$

Besides, to support any $\pi \geq \delta(C-c)$, it is also necessary that

$$\left(\frac{(2n-1)(n+1)}{2n+(n+1)\delta} - \frac{1}{1-\delta}\right)\delta(C-c) \leq \frac{(n-1)((n+1)\delta+1)\delta(C-c)}{(1+\delta)(2n+(n+1)\delta)},$$

which can be verified to be equivalent to

$$n \leq \hat{n}_3.$$

Summing up, we have:

Lemma A3 *For $n \leq \hat{n}_1$, any profit $\pi \in [\delta(C-c), \tilde{\pi}]$ is sustainable by tacit collusion. For $n \in (\hat{n}_1, \hat{n}_3]$, any profit*

$$\pi \in \left[\delta(C-c), \frac{(n-1)(1-\delta)((n+1)\delta+1)\delta(C-c)}{(1+\delta)(2(1-\delta)n^2 - (1+2\delta)n - 1)}\right] \tag{22}$$

is sustainable by tacit collusion.

To support $\pi \in [\tilde{\pi}, \pi^M]$, according to Proposition 1, the most effective collusive prices are

$$(P, p) = (\pi + \tfrac{(2n - \delta(n-1))C + 2n\delta c - (n+1)\delta U}{2n}, \tfrac{(n+1)U + (n-1)C}{2n}).$$

Therefore, the incentive constraint is

$$
\begin{aligned}
\frac{\pi}{n(1-\delta)} &\geq \frac{(2n-1)}{n}\left(\frac{(n+1)U + (n-1)C}{2n} - C\right) + \delta\frac{(C-c)}{1+\delta} \\
&= \frac{(2n-1)(n+1)}{2n^2}(U-C) + \delta\frac{(C-c)}{1+\delta}.
\end{aligned}
\tag{23}
$$

This is easier to satisfy with higher π because the deviation profit is independent of π. In other words, (23) is satisfied for all $\pi \in [\tilde{\pi}, \pi^M]$ if it is satisfied at $\pi = \tilde{\pi}$, i.e.,

$$\frac{1}{n(1-\delta)}\left(U + \delta\frac{(n+1)U + (n-1)C}{2n} - C - \delta c\right) \geq \frac{(1-\delta)(2n-1)(n+1)(U-C)}{2n} + \frac{\delta n(1-\delta)(C-c)}{1+\delta},$$

which can be verified to be equivalent to

$$n \leq \hat{n}_1.$$

Besides, to support any $\pi \leq \pi^M = (1+\delta)U - C - \delta c$, it is necessary that

$$\frac{(1+\delta)U - C - \delta c}{n(1-\delta)} \geq \frac{(2n-1)(n+1)}{2n^2}(U-C) + \delta\frac{(C-c)}{1+\delta}, \tag{24}$$

which can be rewritten as

$$n \leq \frac{(1+\delta)((1+3\delta)(U-C) + 2\delta(C-c)) + \sqrt{(1+\delta)^2((1+3\delta)(U-C) + 2\delta(C-c))^2 + 8(1-\delta)^2(1+\delta)(U-C)(U-C+\delta(U-c))}}{4(1-\delta)(U-C+\delta(U-c))} \equiv \hat{n}_2.$$

It can be verified that the (23) is easier to satisfy for smaller n. This, with the facts that (23) is easier to satisfy for larger π and that $\tilde{\pi} < \pi^M$, implies that $\hat{n}_1 < \hat{n}_2$. Summing up, we have:

Lemma A4 *For $n \leq \hat{n}_1$, any profit $\pi \in [\tilde{\pi}, \pi^M]$ is sustainable by tacit collusion. For $n \in (\hat{n}_1, \hat{n}_2]$, then any*

$$\pi \in \left[\frac{(1-\delta)(2n-1)(n+1)(U-C)}{2n} + \frac{\delta n(1-\delta)(C-c)}{1+\delta}, \pi^M\right] \tag{25}$$

is sustainable by tacit collusion.

Next, we show that $\frac{1}{1-\delta} < \hat{n}_1$ and $\hat{n}_2 < \hat{n}_3$. Recall that the per-generation industry profit $\tilde{\pi}$ can be supported by setting $P = U$ and $p = \frac{(n+1)U + (n-1)C}{2n}$ if and only if $n \leq \hat{n}_1$. At $\pi = \tilde{\pi}$ and $n = \frac{1}{1-\delta}$, the

difference between the equilibrium profit and the deviation profit is

$$\frac{1}{n(1-\delta)}\left(U+\delta\frac{(n+1)U+(n-1)C}{2n}-C-\delta c\right)-\left(\frac{(2n-1)(n+1)}{2n^2}\left(U-C\right)+\delta\frac{(C-c)}{1+\delta}\right)\Big|_{n=(1-\delta)^{-1}}$$

$$=\left(U+\delta\frac{(n+1)U+(n-1)C}{2n}-C-\delta c\right)-\left(\frac{(2n-1)(n+1)}{2n^2}\left(U-C\right)+\delta\frac{(C-c)}{1+\delta}\right)\Big|_{n=(1-\delta)^{-1}}$$

$$=\frac{(1+\delta)(\delta n(n+1)-(n-1))(U-C)+2\delta^2 n^2(C-c)}{2n^2(1+\delta)}\Big|_{n=(1-\delta)^{-1}}$$

$$=\delta\frac{(1+\delta)(U-C)+2\delta(C-c)}{2(1+\delta)}>0.$$

In other words, the per-generation industry profit $\tilde{\pi}$ can be supported among more than $(1-\delta)^{-1}$ firms; so $\hat{n}_1 > (1-\delta)^{-1}$.

Next, $\hat{n}_2 < \hat{n}_3$ is established by the fact that the per-generation industry profit π^M can be supported among $\hat{n}_2$ firms but cannot be supported among $\hat{n}_3 \equiv (1+\delta)/(1-\delta)$ firms, as implied by

$$\frac{\pi}{n(1-\delta)}-\left(\frac{(2n-1)}{n}\left(\frac{(n+1)U+(n-1)C}{2n}-C\right)+\delta\frac{(C-c)}{1+\delta}\right)$$

$$=\frac{(1+\delta)U-C-\delta c}{(1+\delta)}-\left(\frac{\left(2\frac{1+\delta}{1-\delta}-1\right)\left(\frac{1+\delta}{1-\delta}+1\right)(U-C)}{2\left(\frac{1+\delta}{1-\delta}\right)^2}+\delta\frac{(C-c)}{1+\delta}\right)$$

$$=\frac{-\delta(1-\delta)(U-C)}{(1+\delta)^2}<0.$$

Now, we are ready to summarize the characterization of the set of equilibrium profits that tacit collusion can support for the case that $C < U$. By applying Lemmas A2-A4, for all $n \leq \hat{n}_1$, any per-generation industry profit in $[0, \delta(C-c)] \cup (\delta(C-c), \tilde{\pi}] \cup (\tilde{\pi}, \pi^M] = [0, \pi^M]$ can be supported by tacit collusion; this proves part (i) of the theorem. By once again applying Lemmas A2-A4, the set of sustainable per-generation industry profits for $n \in (\hat{n}_1, \hat{n}_2]$ is

$$[0, \delta(C-c)] \cup \left[\delta(C-c), \frac{(n-1)(1-\delta)((n+1)\delta+1)\delta(C-c)}{(1+\delta)(2(1-\delta)n^2-(1+2\delta)n-1)}\right] \cup \left[\frac{(1-\delta)(2n-1)(n+1)(U-C)}{2n}+\frac{\delta n(1-\delta)(C-c)}{1+\delta}, \pi^M\right].$$

This proves part (ii) of the theorem. Similarly, according to Lemmas A2-A4, for $n \in (\hat{n}_2, \hat{n}_3]$, the set of sustainable π is

$$[0, \delta(C-c)] \cup \left[\delta(C-c), \frac{(n-1)(1-\delta)((n+1)\delta+1)\delta(C-c)}{(1+\delta)(2(1-\delta)n^2-(1+2\delta)n-1)}\right].$$

This proves part (iii) of the theorem. Finally, the range of sustainable π as listed in part (vi) of the theorem for $n > \hat{n}_3$ follows immediately Lemma A2. ∎

Proposition A1 *Suppose $p = c$. If $n \leq \frac{1}{1-\delta}$, then any per-generation industry profit $\pi \in [0, \pi^M]$ is sustainable by tacit collusion; if*

$$n \in \left(\frac{1}{1-\delta}, \frac{1}{1-\delta}\frac{U-C+\delta(U-c)}{U-C+\delta(C-c)}\right],$$

28

then any per-generation industry profit $\pi \in [n(1-\delta)(U - C + \delta(C - c)), \pi^M]$ *is sustainable by tacit collusion; otherwise, firms necessarily earn zero profit.*

Moreover, the monopoly profit is sustainable among a larger set of discount factors when each firm monopolizes its aftermarket than when the aftermarkets are competitive if and only if

(i) $C > \overline{C} \equiv (1 - \bar{\gamma})U + \bar{\gamma}c$, *where* $\bar{\gamma} = [9\sqrt{17} + 55]/206 \approx 0.447$, ***or***

(ii) $C \in (c, \overline{C}]$ *and* $n > \hat{n}(C)$, *for some* $\hat{n}(C) \in (2, \infty)$.

Proof of Proposition A1 Knowing that they only have to pay c for the cartridge and are able to gain a surplus of $(U - c)$ in the second period of their market life, new consumers are willing to pay up to $U + \delta(U - c)$ for a printer. As a result, firms may still collude on a printer price $P \in (C, U + \delta(U - c)]$. In this setting, since firms only earn profits from printer sales, the per-generation industry profit becomes $\pi = P - C$. The discounted value of the stream of profits to a firm is $(P - C)/n(1 - \delta)$, where n is the number of firms. Zero-profit pricing means $P = C$.

For a deviating firm to attract new customers, it is necessary for its deviation price to be less than P. Moreover, upon seeing a price cut, rational consumers will anticipate a price war in which printers are sold at $P = C$ and they can enjoy a consumer surplus of $(U - C)$ if they choose not to purchase a printer this period. Therefore, a deviating price P' will attract new customers only if

$$U - P' + \delta(U - c) \geq 0 + \delta(U - C),$$

$$\Leftrightarrow \qquad P' \leq U + \delta(C - c).$$

Summing up, a deviating firm can gain an instantaneous profit arbitrarily close to $(\min\{P, U + \delta(C - c)\} - C)$. Therefore, the condition for the collusive outcome to be sustainable is

$$\frac{P - C}{n(1 - \delta)} \geq \min\{P, U + \delta(C - c)\} - C. \tag{26}$$

For all $\pi = P - C \in (0, U - C + \delta(C - c)]$, $P \leq U + \delta(C - c)$. So once again tacit collusion is sustainable if and only if $n \leq 1/(1 - \delta)$.

For $\pi = P - C \in (U - C + \delta(C - c), U - C + \delta(U - c)]$, $P > U + \delta(C - c)$. So tacit collusion is sustainable if and only if

$$\frac{\pi}{n(1 - \delta)} \geq U + \delta(C - c) - C, \tag{27}$$

which is the easiest to satisfy when $\pi = \pi^M$. Therefore, the necessary condition for sustainability of some profits is

$$n \leq \frac{\pi^M}{(1 - \delta)[U - C + \delta(C - c)]} = \frac{1}{1 - \delta}\frac{U - C + \delta(U - c)}{U - C + \delta(C - c)}. \tag{28}$$

If (28) is satisfied, then (27) can be rewritten to provide the lower bound on the sustainable profit as stated in the proposition.

Now consider how introducing aftermarket competition affects the equipment sellers' ability to tacitly collude. For our purpose, we only compare the firms' abilities to sustain the monopoly profit. Plugging $P = U + \delta (U - c)$ into (26) implies that firms with competitive aftermarkets can sustain the monopoly profit if and only if

$$\frac{U - C + \delta (U - c)}{n (1 - \delta)} \geq U - C + \delta (C - c). \tag{29}$$

Now recall from the proof of Theorem 1 [condition (24)] that the monopoly profit is sustainable among equipment sellers with constrained aftermarket power if and only if

$$\frac{U - C + \delta (U - c)}{n (1 - \delta)} \geq \frac{(2n - 1)(n + 1)}{2n^2} (U - C) + \delta \frac{(C - c)}{1 + \delta}. \tag{30}$$

Define $\gamma = (U - C) / (U - c)$. Further define

$$
\begin{aligned}
F (\delta, n, \gamma) &= \frac{\gamma + \delta}{n (1 - \delta)}, \\
G_{MA} (\delta, n, \gamma) &= \frac{(2n - 1)(n + 1)}{2n^2} \gamma + \frac{\delta}{1 + \delta} (1 - \gamma), \\
G_{CA} (\delta, \gamma) &= \gamma + \delta (1 - \gamma).
\end{aligned}
$$

Therefore, (30) and (29) are equivalent to $F (\delta, n, \gamma) \geq G_{MA} (\delta, n, \gamma)$ and $F (\delta, n, \gamma) \geq G_{CA} (\delta, \gamma)$.

It is obvious that if $U \leq C$, i.e., $\gamma \leq 0$, then $G_{MA} (\delta, n, \gamma) < G_{CA} (\delta, \gamma)$ and thus the monopolist profit is sustainable for a wider range of discount factors with monopolized aftermarkets than with competitive aftermarkets. Therefore, in the remainder of the proof, we only consider the case of $C \in (c, U)$, i.e., $\gamma \in (0, 1)$.

Note that

$$F (0, n, \gamma) = \frac{\gamma}{n} < G_{CA} (0, \gamma) = \gamma < G_{MA} (0, n, \gamma) = \frac{(2n - 1)(n + 1)}{2n^2} \gamma, \tag{31}$$

$$\frac{\partial F (\delta, n, \gamma)}{\partial \delta} = \frac{1 + \gamma}{n (1 - \delta)^2} > 0, \tag{32}$$

and

$$\frac{\partial^2}{\partial \delta^2} G_{MA} (\delta, n, \gamma) = -2 \frac{(1 - \gamma)}{(1 + \delta)^3} < \frac{\partial^2}{\partial \delta^2} G_{CA} (\delta, \gamma) = 0 < \frac{\partial^2}{\partial \delta^2} F (\delta, n, \gamma) = 2 \frac{1 + \gamma}{n (1 - \delta)^3}. \tag{33}$$

Equations (31) and (33) imply that each pair of these curves at most cross each other once in $\delta \in (0,1)$. Since

$$
\begin{aligned}
\lim_{\delta \to 1} F(\delta, n, \gamma) &= \infty, \\
\lim_{\delta \to 1} G_{MA}(\delta, n, \gamma) &= \frac{(2n-1)(n+1)}{2n^2}\gamma + \frac{(1-\gamma)}{2} < \infty, \\
\lim_{\delta \to 1} G_{CA}(\delta, \gamma) &= 1 + \gamma < \infty,
\end{aligned}
$$

there exists unique $\tilde{\delta}_{MA}(n, \gamma) \in (0,1)$ and unique $\tilde{\delta}_{CA}(n, \gamma) \in (0,1)$ at which $F(\delta, n, \gamma)$ intersects with $G_{MA}(\delta, n, \gamma)$ and $F(\delta, n, \gamma)$ intersects with $G_{CA}(\delta, \gamma)$ respectively and (30) is satisfied if and only if $\delta \geq \tilde{\delta}_{MA}(n, \gamma)$ and (29) is satisfied if and only if $\delta \geq \tilde{\delta}_{CA}(n, \gamma)$.

Lemma A5 *Suppose* $\tilde{\delta}_{MA}(n, \gamma) \leq \tilde{\delta}_{CA}(n, \gamma)$. *Then* $\tilde{\delta}_{MA}(n', \gamma) < \tilde{\delta}_{CA}(n', \gamma)$ *for all* $n' > n$. *Suppose* $\tilde{\delta}_{MA}(n, \gamma) \geq \tilde{\delta}_{CA}(n, \gamma)$. *Then* $\tilde{\delta}_{MA}(n', \gamma) > \tilde{\delta}_{CA}(n', \gamma)$ *for all* $n' < n$.

Proof Let $\bar{\delta}(n, \gamma) \in (0,1)$ solve $G_{MA}(\delta, n, \gamma) = G_{CA}(\delta, \gamma)$. It follows from (31) and (33) that $G_{MA}(\delta, n, \gamma) > G_{CA}(\delta, \gamma)$ for $\delta \in (0, \bar{\delta})$ and $G_{MA}(\delta, n, \gamma) < G_{CA}(\delta, \gamma)$ for $\delta \in (\bar{\delta}, 1)$. Suppose F intersects with G_{MA} or G_{CA} at some $\delta < \bar{\delta}$. Since $G_{MA} > G_{CA}$ for $\delta < \bar{\delta}$, F must first intersect with G_{CA} as δ increases. Since they intersect only once, F must intersect with G_{MA} at above G_{CA} which can take place only at some $\tilde{\delta}_{MA} \leq \bar{\delta}$. That F increases in δ and $G_{CA} < G_{MA}$ for $\delta < \bar{\delta}$ imply that $\tilde{\delta}_{CA} \leq \tilde{\delta}_{MA}$. By extending this logic to the other possibility that F intersects with either G_{MA} or G_{CA} at some value larger than $\bar{\delta}$, we can conclude that there are only two possibilities: $\tilde{\delta}_{MA}(n, \gamma) \leq \tilde{\delta}_{CA}(n, \gamma) \leq \bar{\delta}$ or $\tilde{\delta}_{MA}(n, \gamma) \geq \tilde{\delta}_{CA}(n, \gamma) \geq \bar{\delta}$.

Suppose $\tilde{\delta}_{MA}(n, \gamma) \leq \tilde{\delta}_{CA}(n, \gamma) \leq \bar{\delta}$. If $n' > n$, then $F(\delta, n', \gamma) > F(\delta, n, \gamma)$ and $G_{MA}(\delta, n', \gamma) > G_{MA}(\delta, n, \gamma)$. Following the upward shifts of both F and G_{MA}, F intersects with G_{MA} and G_{CA} to the left of $\bar{\delta}$. Therefore, $\tilde{\delta}_{MA}(n', \gamma) < \tilde{\delta}_{CA}(n', \gamma)$.

A similar logic implies that if $\tilde{\delta}_{MA}(n, \gamma) \geq \tilde{\delta}_{CA}(n, \gamma) \geq \bar{\delta}$, then $\tilde{\delta}_{MA}(n', \gamma) > \tilde{\delta}_{CA}(n', \gamma) > \bar{\delta}$ for $n' > n$. This completes the proof of Lemma A5.

When $n = 2$, (30) and (29) are reduced to

$$
\frac{\gamma + \delta}{2(1-\delta)} \geq \frac{9}{8}\gamma + \frac{\delta}{1+\delta}(1-\gamma), \tag{34}
$$

$$
\frac{\gamma + \delta}{2(1-\delta)} \geq \gamma + \delta(1-\gamma). \tag{35}
$$

31

These inequalities can be rewritten as

$$\delta \geq \tilde{\delta}_{MA}(2,\gamma) \equiv \frac{1}{\gamma + 12}\left(\sqrt{36\gamma + 41\gamma^2 + 4} + 2 - 6\gamma\right),$$

$$\delta \geq \tilde{\delta}_{CA}(2,\gamma) \equiv \frac{1}{4(1-\gamma)}\left(\sqrt{8\gamma^2 + 1} + 1 - 4\gamma\right).$$

It can be verified that $\tilde{\delta}_{MA}(2,\gamma) < \tilde{\delta}_{CA}(2,\gamma)$ if and only if $\gamma > \bar{\gamma} \equiv \frac{9\sqrt{17}+55}{206} \approx 0.447$. Applying Lemma A5, we know that $\tilde{\delta}_{MA}(n,\gamma) < \tilde{\delta}_{CA}(n,\gamma)$ for all $n \geq 2$ and $\gamma > \bar{\gamma}$, i.e., $C > \overline{C} \equiv (1 - \bar{\gamma})U + \bar{\gamma}c$.

Suppose $\gamma \leq \frac{9\sqrt{17}+55}{206}$, i.e., $C \leq \overline{C}$. Since

$$\lim_{n \to \infty} G_{MA}(\delta, n, \gamma) - G_{CA}(\delta, \gamma)$$
$$= -\frac{1 - \gamma}{1 + \delta} < 0.$$

Therefore, there exists $\hat{n}(\gamma) > 2$ such that for all $n > \hat{n}(\gamma)$, $\tilde{\delta}_{MA}(n,\gamma) < \tilde{\delta}_{CA}(n,\gamma)$ and for all $n < \hat{n}(\gamma)$, $\tilde{\delta}_{MA}(n,\gamma) > \tilde{\delta}_{CA}(n,\gamma)$.

When $\gamma = 1$, i.e., $C = c$. In this case,

$$G_{MA}(\delta, n, \gamma) - G_{CA}(\delta, \gamma) = \frac{n - 1}{2n^2} > 0.$$

Therefore, $\tilde{\delta}_{MA}(n,\gamma) > \tilde{\delta}_{CA}(n,\gamma)$. This completes the proof that $\tilde{\delta}_{MA} < \tilde{\delta}_{CA}$ if and only if the condition in the proposition is satisfied. ∎

Proposition A2 *Suppose consumers firms are able to offer long-term contracts. Then for all*

$$n \leq \frac{1 + \delta}{1 - \delta},$$

there exists a profitable collusive equilibrium in which firms offer long-term contracts on the equilibrium path and the industry profit is π^M. Moreover

$$\frac{1 + \delta}{1 - \delta} > \hat{n}_2.$$

Proof of Proposition A2. Suppose firms sign long-term contracts with consumers on the equilibrium path, charging them $(1 + \delta)U$ over the consumer's lifetime. Since the contract is binding on both parties, it is equivalent to a bundled contract in which the consumer pays $B = (1 + \delta)U$ up-front and the firm supplies a printer when the consumer is new and commits to deliver a replacement cartridge at no additional charge in the second period of the consumer's market life. This corresponds to a long-term contract with $P = B$ and $p = 0$.

If any firm deviates, the industry reverts to setting $P = p = p^C$. Upon observing a deviation, sophisticated consumers anticipate a price war. Their best outside option of taking the deviator's offer is to abstain from consumption for one period and earn a surplus of $U - p^C$ in the following period. A deviator's offer must provide at least the same surplus. There are two ways the deviator can deviate. First, if the deviator offers a bundle at price B^D, then B^D must satisfy

$$
\begin{aligned}
(1+\delta)\,U - B^D &\geq \delta\left(U - p^C\right)\\
B^D &\leq U + \delta p^C.
\end{aligned}
$$

Since the demands of all consumers in the second period of their market life are already satisfied with a preexisting contract signed in the previous period, the only deviation profit is $B^D - (C + \delta c)$. Second, if the deviator offers a printer-only offer instead, then

$$
(1+\delta)\,U - P^D - \delta p^C \geq \delta\left(U - p^C\right)
$$

must be met. The corresponding deviation profit is $P^D - C + \delta\left(p^C - c\right)$. One can check that both types of deviation lead to the same profit.

Therefore, tacit collusion is sustainable if

$$
\begin{aligned}
B^D - (C + \delta c) &\leq \frac{U - C + \delta\left(U - c\right)}{(1-\delta)\,n}\\
U + \delta\frac{C + \delta c}{1 + \delta} - (C + \delta c) &\leq \frac{U - C + \delta\left(U - c\right)}{(1-\delta)\,n}\\
n &\leq \frac{1+\delta}{1-\delta}.
\end{aligned}
$$

Below we directly verify $\dfrac{1+\delta}{1-\delta} > \hat{n}_2$:

$$
\frac{1+\delta}{1-\delta} > \frac{(1+\delta)((1+3\delta)(U-C)+2\delta(C-c))+\sqrt{(1+\delta)^2((1+3\delta)(U-C)+2\delta(C-c))^2+8(1-\delta)^2(1+\delta)(U-C)(U-C+\delta(U-c))}}{4(1-\delta)(U-C+\delta(U-c))} \equiv \hat{n}_2
$$

$$
\Leftrightarrow \quad \left(\frac{1+\delta}{1-\delta}\left(4\left(1-\delta\right)\left(U - C + \delta\left(U - c\right)\right)\right) - (1+\delta)\left((1+3\delta)\left(U - C\right) + 2\delta\left(C - c\right)\right)\right)^2
$$
$$
> (1+\delta)^2\left((1+3\delta)\left(U - C\right) + 2\delta\left(C - c\right)\right)^2 + 8\left(1-\delta\right)^2\left(1+\delta\right)\left(U - C\right)\left(U - C + \delta\left(U - c\right)\right)
$$

$$
\Leftrightarrow \quad \left(16\delta\left(1 - \delta^2\right)\left(U - C\right)\left(U - C + \delta\left(U - c\right)\right)\right) > 0.
$$

∎

References

[1] Ausubel, Lawrence M. and Raymond J. Deneckere (1987), "One Is Almost Enough for Monopoly," *RAND Journal of Economics*, 18(2), pp. 255-74.

[2] Bajari, Patrick and Lixin Ye (2003), "Deciding Between Competition and Collusion," *Review of Economics and Statistics*, 85(4), pp. 971-989.

[3] Borenstein, Severin, Jeffrey K. Mackie-Mason, and Janet S. Netz (1995), "Antitrust Policy in Aftermarkets," *Antitrust Law Journal*, 63, pp. 455-82.

[4] ———, ———, and ——— (2000), "Exercising Market Power in Proprietary Aftermarkets," *Journal of Economics and Management Strategy*, 9(2), pp. 157-88.

[5] Caillaud, Bernard and Bruno Jullien (2003), "Chicken & Egg: Competition among Intermediation Service Providers," *RAND Journal of Economics*, 34(2), pp. 309–328.

[6] Carlton, Dennis W. (2001), "A General Analysis of Exclusionary Conduct and Refusal to Deal— Why *Aspen* and *Kodak* are Misguided," *Antitrust Law Journal*, 68, pp. 659-83.

[7] ——— and Michael Waldman (2010), "Competition, Monopoly, and Aftermarkets," *Journal of Law, Economics and Organization*, 26(1), pp. 54-91.

[8] Chen, Pei-Yu, and Lorin M. Hitt (2002), "Measuring Switching Costs and the Determinants of Customer Retention in Internet-enabled Businesses: A Study of the Online Brokerage Industry," *Information Systems Research*, 13(3), pp: 255-274.

[9] Chen, Zhiqi and Thomas W. Ross (1999), "Refusals to Deal and Orders to Supply in Competitive Markets," *International Journal of Industrial Organization*, 17(3), pp. 399-417.

[10] ———, ———, and W. T. Stanbury (1998), "Refusals to Deal and Aftermarkets," *Review of Industrial Organization*, 13(1-2), pp. 131-51.

[11] Dana, James Jr. and Yuk-fai Fong (2011), "Long-lived Consumers, Intertemporal Bundling, and Tacit Collusion," *Journal of Industrial Economics*, 59(4), pp. 609-29.

[12] Dutta, Prajit, Alexander Matros, and Jörgen W. Weibull (2007), "Long-run Price Competition," RAND Journal of Economics, 38(2), pp. 291-313.

[13] Ellison, Glenn (2005), "A Model of Add-on Pricing," *Quarterly Journal of Economics*, 120(2), pp. 585-637.

[14] Farrell, Joseph and Carl Shapiro (1988), "Dynamic Competition with Switching Costs," *Rand Journal of Economics*, 19(1), pp.123-37.

[15] Fong, Yuk-fai and Qihong Liu (2011), "Loyalty rewards facilitate tacit collusion," *Journal of Economics & Management Strategy*, 20(3), pp. 739-775.

[16] Gabaix, Xavier and David Laibson (2006), "Shrouded Attributes and Information Suppression in Competitive Markets," *Quarterly Journal of Economics*, 121(2), pp. 505-40.

[17] Grubb, Michael D. (2009), "Selling to Overconfident Consumers," *American Economic Review*, 99(5), pp. 1770-1807.

[18] Grzybowski, Lukasz (2008), "Estimating Switching Costs in Mobile Telephony in the UK," *Journal of Industry, Competition and Trade*, 8(2), pp: 113-132.

[19] Gul, Faruk (1987), "Noncooperative Collusion in Durable Goods Oligopoly," *RAND Journal of Economics.* Summer 1987, 18(2), pp. 248-54.

[20] Harrington, Joseph E. (2008), "Detecting Cartels," *Handbook of antitrust economics*, pp. 213-245.

[21] Hay, George A. (1993), "Is the Glass Half-Empty or Half-Full?: Reflections on the *Kodak* Case," *Antitrust Law Journal*, 62, pp. 177-91.

[22] Kim, Moshe, Doron Kliger, and Bent Vale (2003), "Estimating Switching Costs: the Case of Banking," *Journal of Financial Intermediation*, 12(1), pp: 25-56.

[23] Miao, Chun-Hui (2010), "Consumer Myopia, Standardization and Aftermarket Monopolization," *European Economic Review*, 54(7), pp. 931-946.

[24] Morita, Hodaka and Michael Waldman (2004), "Durable Goods, Monopoly Maintenance, and Time Inconsistency," *Journal of Economics & Management Strategy*, 13(2), pp. 273-302.

[25] ——— and ——— (2010), "Competition, Monopoly Maintenance, and Consumer Switching Costs," *American Economic Journal: Microeconomics*, 2(1), pp. 230–255.

[26] Porter, Robert H. and J. Douglas Zona (1993), "Detection of Bid Rigging in Procurement Auctions," *Journal of Political Economy*, 101(3), pp. 518-538.

[27] ———— and ———— (1999), "Ohio School Milk Markets: An Analysis of Bidding," *RAND Journal of Economics*, 30(2), pp. 263-288.

[28] Shapiro, Carl (1995), "Aftermarkets and Consumer Welfare: Making Sense of *Kodak*," *Antitrust Law Journal*, 63(2), pp. 483-511.

[29] ———— and David T. Teece (1994), "Systems competition and aftermarkets: An economic analysis of *Kodak*," *Antitrust Bulletin*, 39(1), pp. 135-162.

[30] Whinston, Michael D. (1990), "Tying, Foreclosure, and Exclusion," *American Economic Review*, 80(4), pp. 837-59.