Morale and Debt Dynamics

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Abstract

This paper shows that debt undermines relational incentives and harms worker morale. We build a dynamic model of a manager who uses limited financial resources to simultaneously repay a creditor and motivate a worker. If the manager can divert or misuse revenue, then debt makes managers less willing to follow through on promised rewards, leading to low worker effort. In profit-maximizing equilibria, the firm prioritizes repaying its debts, leading to gradual increases in effort and wages. These dynamics can persist even after debts have been fully repaid. Consistent with this analysis, we document that a firm's financial leverage is negatively related to measures of employee morale, wages, and productivity.

JEL Codes: C73, D21, D86, G32. Keywords: Relational Contracts, Productivity, Debt, Morale

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1 Introduction

Firms borrow money to pursue new opportunities and grow, but even a high-return investment is likely to fail unless employees work hard to implement it. A successful manager must therefore motivate her workers even as she repays her debts. In doing so, she faces a serious commitment problem that arises from her *ex post* temptation to steal or otherwise divert revenue to her own ends.[1] Her promises to repay creditors and reward workers must therefore be credible in the context of those ongoing relationships.

This paper shows how debt constrains a manager's relational contracts with her workers, leading to low compensation, low worker morale, and low productivity. We argue that an indebted manager is less willing to follow through on promised rewards to her workers, who are, in turn, less willing to strive for exemplary performance. The result is "bad morale," in the form of low effort that persists as long as the firm has outstanding debt and might persist even longer. To minimize these negative morale effects, the manager optimally prioritizes repaying her debts; as debt is repaid, her promises to workers become more credible and productivity increases. Thus, financial obligations depress morale and lead to productivity dynamics even when bankruptcy is unlikely to actually occur. We complement this theory by documenting that more highly leveraged firms do indeed tend to have lower employee morale, lower wages, and lower productivity.

As an example of how borrowing constrains relational incentives, consider Lincoln Electric's decision to significantly increase its leverage in the early 1990s. At the time, Lincoln Electric was struggling to recover from a devastatingly unprofitable international expansion. The company paid smaller discretionary bonuses to its U.S. workers to prioritize repaying its mounting debts, a decision that damaged employee morale and threatened to undermine its strong relational incentive system. In a radical departure from the company's cooperative culture, Lincoln workers openly voiced their displeasure regarding small bonuses, while

[1] See Aghion and Bolton (1992), Hart (1995), Holmstrom and Tirole (1997), and Shleifer and Vishny (1997) for commitment problems in credit relationships. See Bull (1987), Levin (2003), and Malcomson (2013) for similar commitment problems in agency relationships.

managers expressed fears that the entire incentive system might unravel if they did not make good on promised rewards (Feder (1994); Hastings (1999)). This example is hardly isolated: Bae et al. (2011) and Fahn et al. (2017) document how firms that treat their employees fairly, or that rely on relational contracts, tend to maintain low debt ratios.

We begin by documenting stylized facts about the relationship between debt, wages, morale, and productivity. A fundamental challenge with such an analysis is that most datasets measure neither morale nor non-contractible effort. To overcome this challenge, we employ administrative data from Germany that includes a firm-level survey on employee morale. After controlling for other firm characteristics that might be correlated with both debt and morale, including a self-reported measure of profitability, we show that a firm's leverage is negatively related to morale. Reinforcing this link, we show that leverage increases are correlated with wage and productivity decreases for a large set of European firms. While these regressions do not explicitly address the endogeneity of debt, the documented relationships hold in both the cross section and within firms. These relationships are also heterogeneous in a way that is consistent with our mechanism: they are strongest in labor-intensive industries and in countries with costly contract enforcement.

These facts lead us to explore a theory of how a firm's financial obligations constrain its relational contracts with workers. In our model, a liquidity-constrained manager borrows money from a creditor to fund a strictly positive-return project. The manager then repeatedly motivates a worker to exert effort, using realized profits to repay the creditor and pay wages to the worker. These payments cannot depend on realized output, so the manager can renege on them and instead steal or divert profits. Her promises are therefore credible only if non-payment is punished by the worker, the creditor, or both.

In equilibrium, both repayments and wages are made credible by the threat of future punishments, so the manager will renege on *both* of them if the *total* payment is too onerous. More indebted managers must promise larger repayments to the creditor, which limits the extent to which they can credibly reward the worker for effort. Consequently, relative to financing a project with cash-on-hand, debt depresses both wages and effort.

We show that in profit-maximizing equilibria, effort is decreasing in the sum of the worker's and creditor's continuation payoffs. When the manager first takes on a loan, this *aggregate obligation* is large and effort is low. The manager optimally prioritizes repaying the creditor; as she does so, effort stochastically increases until it converges to a steady-state level that is independent of the initial loan. Wages are at least weakly backloaded while the manager repays her debts. If they are strictly backloaded, then effort depends on both the current *and the past* capital structures of the firm, and debt (temporarily) continues to depress worker morale even after it has been repaid.

We build on this analysis to explore how the worker and manager might collude against the creditor. We show that profit-maximizing equilibria are susceptible to this kind of collusion, as the manager and the worker have an incentive to *jointly* deviate and split the resulting proceeds. To deter collusion, the creditor must threaten to liquidate the firm unless it is promptly repaid. Liquidation is inefficient, so the threat of it further increases the cost of borrowing and creates another reason to backload wages and quickly repay debt.

Our paper identifies a key obstacle to investment: a firm that relies on external funding, rather than cash-on-hand, risks causing lasting harm to morale and productivity. In sharp contrast to the benchmark of Modigliani and Miller (1958), our mechanism suggests that an organization's credit-market decisions are inextricably linked to both its current and future incentive practices.

Related Literature

Our contribution is to study how debt induces morale, wage, and productivity dynamics in firms. To model morale, we draw from the literature on relational contracts (Bull (1987); MacLeod and Malcomson (1989); Levin (2002); Levin (2003)). Our approach is related to Hennessy and Livdan (2009) and Fahn et al. (2019), both of which study how debt affects relational contracts; however, those papers consider stationary equilibria and so do not speak to how wages, morale, or productivity evolve over time. Li and Matouschek (2013), Englmaier and Fahn (2019), and Fuchs et al. (2021) consider the role of credit constraints and investments in relational contracts, but these papers abstract from financing decisions. More broadly, a growing literature studies dynamics in relationships, though without considering credit markets (Fuchs (2007); Board (2011); Halac (2012); Andrews and Barron (2016); Malcomson (2016); Fong and Li (2017); Barron and Powell (2019)).

A vast literature studies the effects of debt on commitment. Debt can serve as a commitment device, preventing managers from pursuing vanity projects (Jensen (1986)) or tying their hands in negotiations with workers (Benmelech et al. (2012)). However, debt entails its own commitment problems, as indebted managers must be given incentives to repay their creditors (Jensen and Meckling (1976); Myers (1977); Townsend (1979); Gale and Hellwig (1985); Stein (2003)). We study how these commitment problems spill over onto management-worker relationships. Our focus is therefore related to Michelacci and Quadrini (2009), but unlike that paper, we focus on morale and productivity dynamics in a setting without commitment. To analyze the morale effects of debt, we extend the techniques from the literature on dynamic debt contracts (e.g., Thomas and Worrall (1994), Albuquerque and Hopenhayn (2004), Clementi and Hopenhayn (2006), DeMarzo and Fishman (2007), He and Milbradt (2016), DeMarzo and He (2020)).

Our empirical results contribute to the burgeoning literature on how credit markets affect organizational outcomes (e.g., Pagano and Volpin (2008) and Matsa (2018)). Research indicates that indebted firms have trouble attracting workers (Brown and Matsa (2016); Graham et al. (2019)), offering generous compensation (Benmelech et al. (2012)), rewarding suppliers for investments (Kale and Shahrur (2007)), and attaining high levels of productivity (Lichtenberg and Siegel (1990); Manaresi and Pierri (2019)) and growth (Lang et al. (1996)). We make three contributions to this literature. First, and most importantly, we identify a key channel through which debt affects morale, wages, and labor productivity. Second, to the best of our knowledge, we are the first to use administrative data on worker morale to proxy for the effectiveness of relational contracts. Finally, while many of the papers above focus on firms near bankruptcy, our analysis shows that debt can substantially depress worker morale even if bankruptcy is unlikely to actually occur.[2]

2 Stylized Facts about Debt and Morale

We motivate our theory by documenting a negative relationship between a firm's leverage and employee morale, wages, and total factor productivity (TFP-R).

In our model, leverage decreases a manager's credibility with her workers, making it harder to reward those workers and leading to lower morale and productivity. Consistent with this mechanism, our empirical results document that debt is negatively correlated with morale (Table 2), worker pay (Table 3), and firm productivity (Table 3). We also show that these correlations are stronger in settings where relational contracts are more important, such as when the contracting environment is unreliable or when labor is an important input (Table 3). Figure 1 illustrates the connection between these empirical results and our theoretical analysis.

Documenting these associations requires a measure of employee morale. While most datasets lack such measures,[3] the German Employment Agency periodically surveys both workers and firms about morale in the workplace. We are not aware of other papers that have used this morale data to proxy for manager-worker relationships.

Table 1 presents descriptive statistics for this data. For workers' assessments of morale, we use the 2012 and 2014 waves of the Linked Personnel Panel (LPP), which contain a series of questions about employees' attitudes towards their jobs. For firms' assessments of worker morale, we use the responses from the IAB Establishment Panel (IAB-EP), which

[2]In the equilibria studied in Section 4, the *threat* of bankruptcy disciplines behavior, but bankruptcy never occurs on the equilibrium path.

[3]There is a large body of literature in organizational behavior that studies the relationship between employee well-being and productivity (Krekel et al., 2019), but few papers study how firm's actions affect morale, and the ones that do typically focus on compensation and benefits programs (Jones et al., 2019).

contains information about firm financials and perceived personnel problems.[4] We use binary indicators for firm reporting whether they face a problem with "motivation in the workplace" or "workforce turnover." Due to data availability, we focus on 2004–2008 in the IAB-EP data. Not every variable is available in every year. We measure debt by the fraction of total investment that is financed using loans in each year, which is available for 2005 and 2008. In Appendix D, we show that other leverage measures lead to similar results. Firms' assessments of workforce motivation and turnover are available for 2004, 2006, and 2008. Appendix B discusses the data in more detail.

[4]Data access for both datasets was provided via on-site use at the Research Data Centre (FDZ) of the German Federal Employment Agency (BA) at the Institute for Employment Research (IAB) and/or remote data execution. DOI: 10.5164/IAB.IABBP9319.de.en.v1; DOI: 10.5164/IAB.LPP1617.de.en.v1.

Table 1: Selected Summary Statistics

Panel A: Summary Statistics for German Administrative Data:

Firm-Level Data	Mean	Median	St Dev	Min	Max
Leverage	15.4	0	30.4	0	100
Leverage>0	0.250	0	0.433	0	1
Problems with Motivation	0.072	0	0.259	0	1
Problems with Turnover	0.029	0	0.168	0	1
Num of Employees	164.4	21	856.2	0	49723
Obs Cross-Section (Lev):	14 625				
Obs Panel (Lev):	19 227				
Worker-Level Data					
Considers Quitting	1.59	1	0.92	1	5
Commitment to Firm	3.72	3.83	0.89	1	5
Job Satisfaction	7.54	8	1.71	1	10
Observations	11 809				

Panel B: Summary Statistics for Amadeus Data

	Mean	Median	Std Dev	Min	Max
Leverage	0.140	0.114	0.112	0	0.553
ΔLeverage	-0.0024	-0.0024	0.0416	-0.1314	0.1472
Number of employees	140.18	90	135.36	15	775
Total assets (Mln Euro)	45.7	21.7	78.2	0.4	2756.6
ΔTFP-R (GNR)	0.014	0.0173	0.242	-7.176	7.211
Observations	127703				

Panel A presents summary statistics from LPP and IAB-EP data. Statistics for firm-level variables refer to the full dataset which contains values from 2004-2008 period. For panel regressions, only 2 observations per firm are used (2005/2006 and 2008). For cross-sectional regressions, one observation per firm - averaging values from the whole period - is used. For worker-level data, summary statistics come from the pooled cross-section of workers from two waves of LPP survey. For firm-level data, the number of observations is the number of non-missing leverage observations; for worker data, it is the number of observations that are merged with our firm-level leverage measure. "Problems with Motivation/Turnover" are binary indicators. "Considers Quitting" and "Commitment to the Firm" are measured on a discrete scale from 1 to 5. "Job Satisfaction" is measured on a scale from 1 to 10. Panel B presents summary statistics from Amadeus data. The sample contains all "large" and "very large" firms, i.e. those with more than 150 employees, operating revenue higher than 10 million EUR or total assets of at least 20 million EUR in the *last* available year (note that these variables can have lower values in earlier years). Observations come from 1985-2017, but each firm has at most 10 observations, usually from 2006/07-2016/17.

Panel A of Table 2 documents firm-level regressions of morale on leverage. Columns (1)–(2) and (5) study morale for a cross-section of firms, with one observation per firm that equals an average of the IAB-EP data observed between 2004 and 2008. Our specification is

$$Morale_i = \beta_0 + \beta_1 Lev_i + \boldsymbol{\beta X}_i + \epsilon_i, \tag{1}$$

where $Morale_i$ is a firm-reported binary measure of employee morale, as measured by motivation problems in columns (1) and (2) and turnover problems in column (5). The table

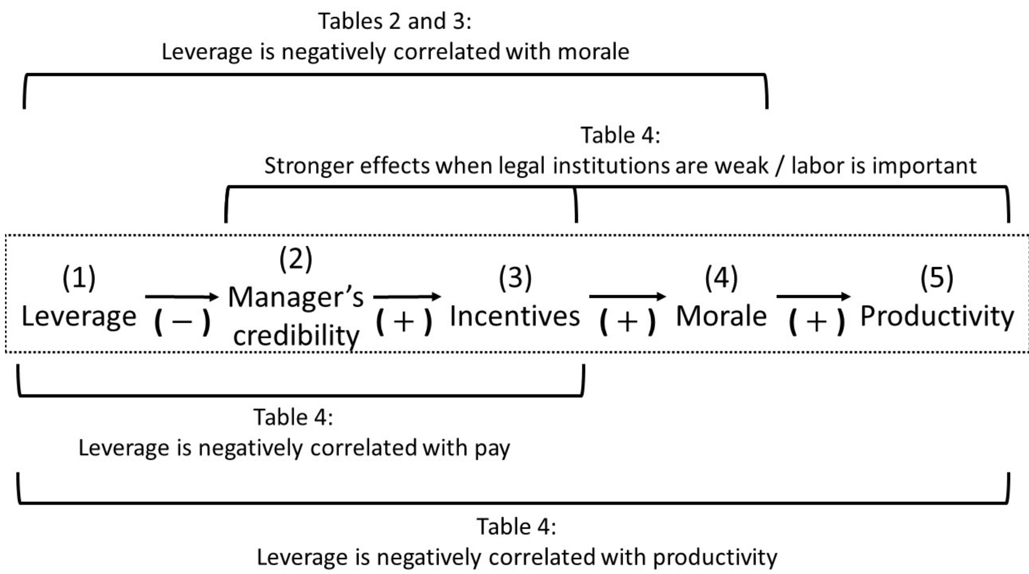

Figure 1: An overview of our mechanism with references to the corresponding empirical results.

description has more information about these variables. Lev_i is a measure of leverage, and $\boldsymbol{X_i}$ is a vector of controls that include industry fixed effects, firm size quintiles, firm age (with an indicator for missing age data), and a self-reported measure of firm profitability.[5] The unit of observation, i, is a firm in all three columns.

Columns (3)–(4) and (6) of Panel A in Table 2 give results for the panel regression:

$$Morale_{it} = \beta_0 + \beta_1 Lev_{it} + \boldsymbol{\beta X_{it}} + \xi_i + \epsilon_{it}, \tag{2}$$

where ξ_i represents firm fixed effects. We include two waves, $t \in \{2005/2006, 2008\}$, where 2005/2006 means the leverage measure from 2005 and the morale measure from 2006. These panel regressions otherwise include the same variables as the cross-sectional regressions.[6]

The first four columns of Panel A in Table 2 illustrate that higher leverage at a firm is correlated with more motivational problems at that firm. For example, column (2) says

[5]Appendix D includes regressions that interact firm profitability and debt. The coefficients on these interaction terms suggest that the negative relationship between debt and morale holds regardless of a firm's profits.

[6]The number of observations in the cross-sectional regressions is lower than in the panel regressions, because the cross-sectional regressions include only one observation per firm. Similar results hold in the pooled cross-sectional regression.

that a one standard deviation increase in leverage is associated with a 1% increase (30.4% x 0.034) in the probability that a firm reports problems with motivating employees, which constitutes 14% of the base probability. The coefficients in cross-sectional regression, (1)–(2), are highly significant, while the coefficient in our panel regression, (3), is marginal (significant at 15%). Given that our panel is very short, column (4) substitutes our leverage measure with an indicator that equals one if the firm uses any debt to finance its investment. This binary measure of leverage is significantly and positively related to the probability that the firm reports problems with motivating workers. Columns (5) and (6) give the corresponding cross-sectional and panel regressions when the dependent variable is an indicator that equals one if the firm reports problems with workforce turnover. Both the cross section and the panel regression exhibit a significant, positive relationship between leverage and problems with worker retention.

We complement these results with data on employees' assessments of their own motivation. Panel B of Table 2 considers the following specification:

$$Morale_{j,i} = \beta_0 + \beta_1 Lev_i + \boldsymbol{\beta X}_{j,i} + \epsilon_j, \tag{3}$$

where i denotes a firm and j denotes a worker. In the basic specification, the vector $\boldsymbol{X_{j,i}}$ includes the worker's sex and age, while the specification with additional controls includes the worker's education, income, type of contract with the firm, and the firm's profitability, age, and average wage. The table description provides details about construction of workers' morale measures. Columns (1) and (2) show that workers at more highly leveraged firms are more likely to frequently consider leaving their jobs, and columns (3) and (4) show that these workers are less likely to report high commitment to the firm. Columns (5) and (6) document a similar correlation between leverage and job satisfaction. Overall, these regressions are consistent with evidence in Panel A.

Our theory says that in addition to depressing morale, leverage affects worker pay and firm productivity. In Appendix D, we report productivity results using German administrative

Table 2: Firm Debt and Worker Motivation and Turnover

Panel A: Firm-Level Analysis

	Cross-section		Panel		Cross-Section	Panel
	Problems with Motivation				Problems with Turnover	
	(1)	(2)	(3)	(4)	(5)	(6)
Leverage	0.047***	0.034***	0.020+		0.011*	0.0235**
	(0.008)	(0.009)	(0.014)		(0.006)	(0.010)
Leverage>0				0.0224**		
				(0.0088)		
N	14 124	9 677	17 522	17 522	9 677	17 522
Fixed Effects	Industry	Industry	Firm	Firm	Industry	Firm
Add. Controls	-	✓	✓	✓	✓	✓

Panel B: Worker-Level Analysis

	Seriously Considers Quitting		High Commitment to Firm		Job Satisfaction	
	(1)	(2)	(3)	(4)	(5)	(6)
Leverage	0.0573**	0.0422**	-0.0838***	-0.0664***	-0.0529***	-0.0407**
	(0.0224)	(0.0217)	(0.0238)	(0.0247)	(0.0185)	(0.0185)
N	7 637	7 399	7 647	7 408	7 645	7 407
Add. Controls		✓		✓		✓

Panel A shows the regression of firms' declarations about problems with the workforce on measures of debt. Columns 1, 2, and 5 show cross-sectional regressions using averages from the 2004–2008 period. Columns 3, 4 and 6 use panel data with two observations per firm. The dependent variable in columns 1-4 is a binary indicator for "problems with workforce motivation", as declared by the firm. The dependent variable in columns 5-6 is a similar indicator for "problems with workforce turnover". The questions we use to measure staffing problems were asked in 2004, 2006, and 2008. Cross-sectional columns use averages of these 3 values, while panel waves use values from 2006 and 2008. The independent variable in the first row is share of debt in the investment volume, which is available in 2005 and 2008. In the panel columns, these two values are used for two waves, while in the cross-sectional columns we use their average. The independent variable in the second row is a binary indicator for the average share of debt exceeding zero.Controls include either firm fixed effects (FE) or industry FE (using the German Employment Agency classification, which roughly corresponds to 2 digit SIC codes) and indicators for five quintiles of total employment. Additional controls include five indicators for each firm's self-reported profitability, its type of management, the growth in its employment, firm age (and an indicator for age being missing), and its average wage. In the panel columns, we add year FE and drop management type, because it is absorbed by firm FE. Standard errors, in panel regression columns clustered on the firm level, are shown in parentheses.

Panel B shows the results of the regression of workers' assessments of their morale on measures of their employer's debt. All regressions are cross-sectional, use the average of workers' answers in waves 2012 and 2014 of the LPP data, and merge those answers with the 2008 measure of the firms' share of debt in investment expenditures from the IAB-EP data. The dependent variable in columns 1-2 is an indicator for a response to the question "How often have you thought about changing your job in past 12 months?" on a 5-point scale being equal or above 3, where the possible answers are: never (1), a few times a year (2), a few times a month (3), every week (4), and every day (5). The dependent variable in columns 3-4 is an indicator for an average of six prompts that measure a worker's commitment to the firm on a 5-point scale being above 3. The prompts are: I am happy to spend rest of my life with the firm; the firm has a great deal of personal meaning for me; the firm's problems are my own; I feel a sense of belonging; I feel emotionally attached to the firm; and I feel part of the family at my firm. The dependent variable in columns 5–6 is an indicator for employee's job satisfaction being equal or above 6 on 10-point scale. The independent variable is the share of debt in the firm's investment volume in 2008. Each column controls for worker's age and sex. Our additional controls include the worker's educational and income levels, the average wage at the firm, the type of the employee's contract (fixed term or permanent), firm age (and an indicator for age being missing), and self-reported measure of firm profitability. Standard errors are shown in parentheses and clustered on the firm level. (***) denotes significance at 1% level, (**) at 5%, (*) at 10% and (+) at 15% level.

data that are consistent with our model. However, the short panel in this data limits our ability to measure capital input and explore productivity dynamics. Therefore, our main analysis of productivity and wages uses data from Amadeus, which has a longer panel with more detailed financial data and larger number of European firms.[7] Our sample covers 16 countries and almost 25,000 public and private manufacturing firms, with up to 10 years of data per firm between 1985–2017.[8]

We measure borrowing by the ratio of non-current liabilities to total assets (book leverage); unlike market leverage, variation in this measure does not directly depend on market beliefs about current or future profits.[9] We proxy for productivity using revenue total factor productivity (TFP-R), which we calculate using the method of Gandhi et al. (2020). This measure has the advantage of controlling for the endogenous choice of inputs, although note that TFP-R reflects both a firm's productive efficiency and the degree of market competition.[10] Appendix B provides additional details about the data and TFP calculations.

Table 3 presents the results of this analysis. Columns (1) and (2) regress wages changes on changes in both current- and previous-year leverage, while columns (5) and (6) use productivity as dependent variable:

$$\Delta Y_{it} = \beta_1 \Delta Lev_{it} + \beta_2 \Delta Lev_{i(t-1)} + \beta_3 X_{it} + \mu_i + \psi_t + \epsilon_{i,t}. \tag{4}$$

Here, ΔY_{it} is the wage or TFP-R change from year $t-1$ to year t. Similarly, $\Delta Lev_{i,t}$ is the change in financial leverage from $t-1$ to t. We include both firm and year fixed effects, so our regression exploits within-firm changes in productivity and leverage while controlling for year-specific aggregate shocks. We also run an alternative specification that

[7] Legal limitations prevent us from performing a unified analysis with both German administrative data and Amadeus data.

[8] Note that this time frame includes a large economic shock in the form of the 2008-2009 financial crisis. In contrast, our morale results (Tables 2 and 3) use data from before and after the crisis.

[9] Since future profits matter for the relational contract between managers and workers, we use book leverage in order to disentangle the effects of debt from the effects of shocks to future profitability.

[10] In unreported regressions, we re-run this analysis with measures of TFP-R calculated from both OLS regressions and the method of Levinsohn and Petrin (2003). Both of these alternative approaches produce results similar to those reported here.

includes industry-year fixed effects. Additional controls, $X_{i,t}$, include changes in the number of employees, firm age, and total fixed assets.[11] Standard errors are clustered by NACE2 industry code.

We find that increases in leverage are significantly correlated with decreases in contemporaneous wages. Lagged leverage changes are also negatively correlated with wages.

Increases in firm leverage are also correlated with decreases in productivity. This relationship is highly significant and economically substantial: one standard deviation increase in contemporary leverage is correlated with a decrease in TFP-R equal to 11.5–13.4% of the median within-firm standard deviation.[12] Lagged increases in leverage are also correlated with decreases in productivity. This correlation between *past* leverage and *current* productivity is consistent with our theoretical result that debt can have lingering effects on morale.

Finally, we explore how the relationship between debt, wages, and productivity varies with the institutional context and the role of labor in production. The mechanism outlined in Figure 1 hinges on a commitment problem: managers can renege on wages and debt repayments. This commitment problem arises because output-contingent contracts cannot be enforced; thus, it is plausibly more severe, and so the relationships between debt, wages, and productivity are stronger, in countries with costlier contract enforcement. Columns (3) and (7) of Table 3 run a version of equation (4) that includes interactions between current and lagged leverage and a country-level measure of the cost of contract enforcement from the World Bank's Doing Business survey.[13] We use this measure as a proxy for the quality of contract enforcement (see, e.g., Fahn et al. (2017)). Consistent with our mechanism, we find that leverage increases are more strongly correlated with wage and productivity decreases in countries where formal contracts are costlier to enforce.

[11] Our main results weight observations equally; we find similar results if we weight by the number of employees, total assets, or gross output, though $\Delta Lev_{i,t-1}$ is not always significant.

[12] The standard deviation of log-change in leverage in our final sample is 0.044, while the within-firm standard deviation of TFP changes for the median firm equals 0.156.

[13] Our main results use a binary indicator of an above-median cost of contract enforcement, but the analysis is robust to using a continuous measure instead.

Table 3: Firm Debt, Wages and Productivity

	ΔL(Wage)				ΔTFP			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
ΔLeverage	-0.111***	-0.086***	-0.006	-0.045**	-0.478***	-0.410***	-0.297***	-0.297***
	(0.0141)	(0.0164)	(0.0190)	(0.0164)	(0.0760)	(0.0665)	(0.0434)	(0.0617)
ΔLeverage (t-1)	-0.031**	-0.022**	-0.063***	-0.068***	-0.102***	-0.066***	-0.129***	-0.103***
	(0.0123)	(0.0104)	(0.0151)	(0.0154)	(0.0210)	(0.0114)	(0.0333)	(0.0211)
ΔLeverage × Costly Enforcement			-0.179***				-0.327***	
			(0.0325)				(0.0961)	
ΔLeverage (t-1) × Costly Enforcement			0.004				0.045	
			(0.0153)				(0.0345)	
ΔLeverage × Labor Intensive				-0.102***				-0.306**
				(0.0255)				(0.108)
ΔLeverage (t-1) × Labor Intensive				0.0144				-0.001
				(0.0210)				(0.0375)
N	127703	127703	127604	127703	127703	127703	127604	127703
Firm FE	✓		✓	✓	✓		✓	✓
Year FE	✓		✓	✓	✓		✓	✓
Industry X Year FE		✓				✓		

The table shows the regression of wages and productivity on financial leverage using firm-level data from Amadeus. The dependent variable in columns 1–4 is the change in the log of average wages, which is calculated as the ratio of total compensation costs to the number of employees. The dependent variable in columns 5–8 is the change in TFP-R, which is calculated using method developed by Gandhi et al. (2020). We measure Leverage as the ratio of non-current liabilities to total assets (book value). The variable 'Costly Enforcement' is an indicator that equals 1 if the country-level measure of cost of contract enforcement (from the Doing Business Survey) is above the median, and equals 0 otherwise. The variable 'Labor Intensive' is an indicator that equals 1 if the industry share of labor input in its OLS-based production function is above the country-wide median, and equals 0 otherwise. Each column includes total fixed assets, number of employees, firm age (and an indicator for age being missing), and a proxy for profitability (value added to assets, and an indicator for this ratio being missing) as additional controls. We cluster standard errors at the industry level. These standard errors are displayed in the parentheses. (***) denotes significance at 1% level, (**) at 5%, (*) at 10% and (+) at 15% level.

Since our mechanism operates through worker effort, it is more relevant when workers are more important to production. To test this idea, we develop a country-industry level proxy for the importance of labor in production. For each country-industry combination, we run an OLS estimation of a standard Cobb-Douglas production function to recover estimated coefficients on labor and capital. Then, we define a firm-country level indicator variable, "labor intensity," which equals 1 when the estimated coefficient on labor for an industry is above the median coefficient across all industries in that country.

Columns (4) and (8) of Table 3 interact this measure of labor intensity with leverage.[14] As predicted by our mechanism, this interaction term is negative and significant, suggesting that the association between leverage, compensation, and productivity is stronger in more labor-intensive industries.

Appendix D reports additional results that explore heterogeneity across industries and profitability levels, as well as the possibility that leverage has non-linear effects. This appendix also presents robustness tests with alternative measures of leverage, which largely reinforce the idea that higher debt is correlated with lower worker morale, wages, and productivity.[15] Interestingly, we find evidence that higher debt is linked to lower morale even among highly profitable firms. Thus, consistent with our mechanism, these relationships hold even for firms that are otherwise financially healthy.

Since we do not have exogenous variation in leverage, we cannot identify the causal effect of debt. Yet, our results are obtained controlling for firm size, age, and profitability, as well as for firm fixed effects in the panel specification, thus alleviating some of the most pressing endogeneity concerns. Our regressions that interact debt with cost of contract enforcement or labor intensity also produce results consistent with our proposed mechanism. As reported in Appendix D, Granger causality tests show that leverage changes Granger-cause productivity changes, while there is marginally significant Granger-causality in the other direction. Taken

[14] We do not include this indicator as a separate regressor because it is absorbed by firm fixed effects.

[15] Some of our results lose statistical significance, although even in those cases, the point estimates are still typically correctly signed.

together, these results suggest that financial obligations limit a firm's ability to motivate its workers.

3 Model

We now develop a dynamic model to explore how debt affects the dynamics of morale.

Consider a game played by a manager ('she'), a creditor ('it'), and a worker ('he'), who share a common discount factor $\delta \in [0, 1)$. The manager needs a loan of size $L > 0$ to finance a positive-return project. She writes a contract to borrow money from a deep-pocketed creditor. The contract specifies a liquidation probability in each period, $l(\cdot) \in [0, 1]$, contingent on the history of repayments to the creditor.

The creditor accepts or rejects this contract. If it accepts, then it pays L, the project is funded, and the game continues; otherwise, the game ends and players earn 0.

If the project is funded, then the manager and the worker interact repeatedly. All actions are publicly observed, but the only contractible variables are repayments to the creditor (r_t) and a public randomization device that is realized at every stage of each period. In particular, formal contracts cannot condition on the firm's output, creating scope for relational contracts to improve outcomes.

If the creditor agrees to the contract, then the manager and worker play the following stage game in each period $t \in \{0, 1, ...\}$:

1. The worker chooses **effort,** $a_t \in \mathbb{R}_+$.

2. A **productivity state,** $\theta_t \in \{0, 1\}$, is realized according to the distribution $\Pr\{\theta_t = 1\} = p$.

3. **Output** $y_t = \theta_t a_t$ is realized.

4. The manager pays a **wage** to the worker, $w_t \geq 0$, and a **repayment** to the creditor, $r_t \geq 0$, where $w_t + r_t \leq y_t$.

5. The project is **liquidated** with probability $l_t \equiv l(h_c^t)$, where h_c^t denotes the history of repayments to the creditor, $h_c^t = (r_0, ..., r_t)$.

If the project has not yet been liquidated, then the manager's and worker's period-t payoffs are $\pi_t = y_t - w_t - r_t$ and $u_t = w_t - c(a_t)$, respectively, where $c(\cdot)$ is non-negative, strictly increasing, strictly convex, and satisfies $c(0) = c'(0) = 0$ and $\lim_{a\to\infty} c'(a) = \infty$. The creditor's period-$t$ payoff is r_t. Following liquidation, the game ends and all players earn 0. The manager's, worker's, and creditor's normalized discounted continuation payoffs in period t are $\Pi_t = \sum_{t'=t}^{\infty} \delta^{t'-t}(1-\delta)\pi_{t'}$, $U_t = \sum_{t'=t}^{\infty} \delta^{t'-t}(1-\delta)u_{t'}$, and $K_t = \sum_{t'=t}^{\infty} \delta^{t'-t}(1-\delta)r_{t'}$, respectively.

In this model, the agent's "morale" is captured by his effort. **First-best effort**, a^{FB}, satisfies $c'(a^{FB}) = p$. We assume throughout that $pa^{FB} - c(a^{FB}) > L$, so the project has strictly positive net-present value if the worker exerts first-best effort.[16] Define $h^t \in \mathcal{H}^t$ as a history in period t. We consider **profit-maximizing** subgame-perfect equilibria (SPE), which maximize the manager's *ex ante* expected payoff among SPE.

The manager has limited commitment, because she can refuse to pay r_t or w_t and suffer no worse punishment than losing her continuation payoff. We model this situation by assuming that the formal contract cannot depend on y_t, which means that the manager can always renege on her promises by acting as if $y_t = 0$. In practice, output is non-contractible because managers can divert profits in undetectable ways, for instance, by diverting money into privately beneficial but unprofitable ventures (see, e.g., Hart (1995); Holmstrom and Tirole (1997); Albuquerque and Hopenhayn (2004); and DeMarzo and Fishman (2007)). Thus, financial instruments can condition on *reported* profits but not *true* profits.[17]

As discussed in Section 6, while we model the firm's financial obligations as debt, similar commitment problems can arise from equity and other financial contracts. We assume that

[16] Note that δ does not appear in this expression, because first-best profits are already measured in net present-value terms: $\sum_{t=0}^{\infty} \delta^t(1-\delta)(pa^{FB} - c(a^{FB})) = pa^{FB} - c(a^{FB})$.

[17] We could allow the manager to make a cheap-talk report about profits without affecting our analysis. For example, the following model leads to identical results: the manager makes a report, $\hat{y}_t$, after y_t is realized in each period. The formal contract can condition on $\hat{y}_t$, but the manager can freely report any $\hat{y}_t \leq y_t$ and steal $y_t - \hat{y}_t$.

the manager keeps the entire output when she reneges on a payment, but our takeaways would still hold if she kept only a fraction of output. Similarly, we could assume that liquidation generates a strictly positive scrap value without affecting the analysis. The assumption that $l(\cdot)$ can depend on repayments to the creditor but not on wages to the worker is irrelevant in Section 4, but it is important in Section 5, because it allows the manager and worker to "secretly" pay one another when colluding. The assumption that the manager cannot borrow or save money between periods is discussed in Section 6.

4 Financial Constraints and Morale

This section analyzes effort, productivity, and wage dynamics in profit-maximizing equilibria. To do so, we formulate the equilibrium payoff frontier as a dynamic program, consider two benchmarks that highlight the role of the manager's commitment problem, and then prove our main results. All proofs are in Appendix A.

The worker or the creditor can punish the manager equally harshly for not paying them, the former by choosing $a_t = 0$ in each period and the latter by specifying in the contract that non-payment results in the liquidation of the firm. However, since output is not contractible, only the worker can condition his punishment on realized output. Consequently, it is efficient for the worker to punish *any* deviation by the manager, including deviations in r_t.[18] Thus, it is without loss to set $l(\cdot) \equiv 0$ and assume that players earn 0 following any deviation in a profit-maximizing equilibrium.

Given these punishments, we can derive the dynamic program for a profit-maximizing equilibrium. Let

$$E = \{(U, \Pi) | \exists K \in \mathbb{R} \text{ s.t. } \exists \text{ SPE with worker, manager, and creditor payoffs } (U, \Pi, K)\}$$

[18] Indeed, without the worker, profit-maximizing equilibria would necessarily entail on-path liquidation. See, e.g., Clementi and Hopenhayn (2006).

be the set of the worker's and the manager's equilibrium continuation payoffs. Given $(U, \Pi) \in E$, define $K(U, \Pi)$ as the creditor's maximum equilibrium continuation payoff if the worker and manager earn U and Π, respectively. Noting that $w_t = r_t = 0$ if $\theta_t = 0$, denote $w_t \equiv w$ and $r_t \equiv r$ if $\theta_t = 1$. Let (U_L, Π_L) and (U_H, Π_H) be the worker's and manager's continuation surpluses from period $t+1$ onwards if $\theta_t = 0$ or $\theta_t = 1$, respectively.

Equilibrium strategies must satisfy several sets of constraints. Consider period $t \geq 0$ of an equilibrium in which the manager and worker earn Π and U, respectively. **Promise-keeping constraints** require these parties to actually earn these payoffs:

$$U = (1-\delta)(pw - c(a)) + \delta(pU_H + (1-p)U_L) \text{ and} \tag{PK-A}$$

$$\Pi = (1-\delta)p(a - w - r) + \delta(p\Pi_H + (1-p)\Pi_L). \tag{PK-P}$$

The **incentive constraint** ensures that the worker is willing to choose $a \geq 0$. Since he earns U from doing so and no more than 0 from deviating, this constraint is simply

$$U \geq 0. \tag{IC}$$

The **dynamic enforcement constraint** captures the manager's commitment problem. If $\theta = 1$, then she must prefer paying w and r to paying nothing and earning 0 continuation payoff:

$$\delta \Pi_H \geq (1-\delta)(w + r). \tag{DE}$$

Limited liability constraints require r and w to be non-negative and less than output:

$$r \geq 0; \quad w \geq 0; \quad r + w \leq a. \tag{LL}$$

Finally, continuation payoffs must be attainable in equilibrium:

$$(U_H, \Pi_H), (U_L, \Pi_L) \in E. \tag{CE}$$

	With (DE)	Without (DE)
With (IC)	**Productivity / morale dynamics** **morale and productivity increase as debt decreases** **Some positive-return projects unfunded**	First-best is attained No dynamics: $a_t = a^{FB}$ in all t Positive-return projects always funded
Without (IC)	First-best effort, under-investment No dynamics: $a_t = a^{FB}$ in all t Some positive-return projects unfunded	First-best is attained No dynamics: $a_t = a^{FB}$ in all t Positive-return projects always funded

Figure 2: A summary of the benchmarks and main results.

Given (U, Π), the equilibrium that maximizes the creditor's payoff solves

$$\begin{aligned} K(U,\Pi) \equiv \quad & \max_{r,w,a,U_H,U_L,\Pi_H,\Pi_L} \quad (1-\delta)pr + \delta\left(pK(U_H,\Pi_H) + (1-p)K(U_L,\Pi_L)\right) \\ & \text{subject to} \qquad \text{(PK-A),(PK-P),(IC),(DE),(LL),(CE).} \end{aligned} \tag{5}$$

The creditor is willing to loan the manager funds as long as at the start of the game, continuation utilities, $(U_0, \Pi_0) \in E$, satisfy $K(U_0, \Pi_0) \geq L$. Therefore, characterizing equilibrium dynamics amounts to characterizing the equilibrium payoff frontier $K(\cdot, \cdot)$.

The two key incentive problems are represented by (IC) and (DE), which give the worker and the manager the incentive to follow through on a and (w, r), respectively. We begin by studying two benchmarks, each of which removes one of these constraints. Figure 2 summarizes these benchmarks.

The first benchmark eliminates the manager's commitment problem and so removes (DE) as a constraint. To accomplish this, we make output contractible, so that the formal contract can specify w_t and r_t as a function of the history of realized outputs, $\{y_{t'}\}_{t'=0}^{\infty}$. Profit-maximizing equilibria attain first-best in this benchmark.

Proposition 1 *Suppose output is contractible. In any profit-maximizing equilibrium, the project is funded and $a_t = a^{FB}$ in all $t \geq 0$.*

If output is contractible, then the manager can commit to repay the creditor L and exactly compensate the worker for his efforts. This contract makes the manager the residual claimant on output. The project has strictly positive net-present value, so the profit-maximizing contract funds the project and then induces first-best effort in each period.

The second benchmark eliminates the agent's incentive problem by removing (IC) as a constraint. The following **bilateral game** does so while preserving the worker's ability to punish the manager following a deviation. Suppose that whenever $\theta_t = 1$, the manager is *committed* to pay the following wage to the worker:

$$w_t = \begin{cases} \frac{c(a_t)}{p} & a_t \leq a^{FB} \\ 0 & a_t > a^{FB}. \end{cases}$$

When $\theta_t = 0$, $w_t = r_t = 0$ as in the baseline model. This benchmark eliminates the commitment problem with the worker by ensuring that he is exactly compensated for choosing any $a_t \in [0, a^{FB}]$. In particular, he is willing to choose $a_t = a^{FB}$ on the equilibrium path and $a_t = 0$ following a deviation. The manager can still renege on the creditor, which is the sense in which the commitment problem is "bilateral" in this benchmark.

We show that this bilateral commitment problem drives a wedge between the value that is created by the project and the value that can be pledged to the creditor. Conditional on the project being funded, however, the manager is residual claimant on output and so $a_t = a^{FB}$ in each period.

Proposition 2 *In any profit-maximizing equilibrium of the bilateral game, the project is funded if and only if* $L \leq \frac{\delta p}{1-\delta+\delta p}\left(pa^{FB} - c(a^{FB})\right)$. *If the project is funded, then* $a_t = a^{FB}$ *for all* $t \geq 0$.

The manager's stage-game payoff is $pa_t - c(a_t) - r_t$ in the bilateral game. Setting $a_t = a^{FB}$ in each period is therefore optimal, since it maximizes both the manager's payoff and the amount she would lose from reneging on r_t. Satisfying (DE) requires $\Pi_H > 0$ whenever $r > 0$, so the creditor can be pledged only a fraction $\delta p/(1 - \delta + \delta p)$ of the project's proceeds.

In both of these benchmarks, profit-maximizing equilibria are stationary and entail $a_t = a^{FB}$ in every period. We now turn to our main analysis and show that together, (IC) and (DE) lead to effort dynamics. Our first step is to prove that effort and the manager's maximum equilibrium payoff depend on the sum of the creditor's and worker's payoffs, $K(U,\Pi) + U$, but not on how this sum is split. Define

$$\tilde{U}(\Pi) = \max\left\{U | (U,\Pi) \in E\right\}$$

as the worker's maximum equilibrium continuation utility given Π.

Lemma 1 *For any* $(U,\Pi) \in E$,

$$K(U,\Pi) + U = \tilde{U}(\Pi).$$

Lemma 1 implies that profit-maximizing equilibria depend only on the *sum* of the creditor's and worker's continuation payoffs. This result holds because, fixing $w+r$, we can adjust w and r without violating (DE). We show that it is possible to do so in a way that also respects (IC), which means that transfers can freely divide the sum, $K(U,\Pi) + U$, between the worker and the creditor.[19]

[19] A version of this lemma holds in more general settings with multiple workers and creditors, provided that effort is still observable. Consequently, similar dynamics would arise in richer environments.

Lemma 1 allows us to characterize equilibrium effort dynamics using techniques adapted from Albuquerque and Hopenhayn (2004). Define

$$a_{max} = \arg\max_{a \geq 0} \left\{ pa - c(a) | \frac{c(a)}{p} \leq \frac{\delta}{1-\delta}(pa - c(a)) \right\}. \tag{6}$$

Intuitively, a_{max} is the most efficient effort that could be attained in equilibrium if the manager did not have to borrow any money (i.e., if $L = 0$). The conditioning inequality in (6) is simply (DE), with $w = \frac{c(a)}{p}$, $r = 0$, and $\Pi_H = pa - c(a)$. If the manager had enough cash-on-hand to finance the project internally, then $a_t = a_{max}$ in every period of any profit-maximizing equilibrium. It turns out that a_{max} is an upper bound on effort in profit-maximizing equilibria.

We now present our main result, which shows that relative to the benchmark of using cash-on-hand ($L = 0$), debt depresses effort and leads to dynamics in profit-maximizing equilibria.

Proposition 3 *If there exists a $\Pi_0 > 0$ such that $K(0, \Pi_0) = L$, then the project is funded in any profit-maximizing equilibrium. In any such equilibrium, let Π be the manager's continuation payoff in period t. Then:*

1. *$a_t = a^*(\Pi)$, where*
$$a^*(\Pi) = \min\left\{ a_{max}, \frac{(1-\delta(1-p))}{(1-\delta)p}\Pi \right\}.$$

2. *If $0 < a_t < a_{max}$, then $\Pi_L = \Pi < \Pi_H$.*

3. *If $\theta_t = 0$, then $a_{t+1} = a_t$. If $\theta_t = 1$, then: (i) $a_{t+1} > a_t$ unless $a_t = a_{max}$, in which case $a_{t+1} = a_{max}$, and (ii) $w_t + r_t = a_t$ unless $a_{t+1} = a_{max}$. Moreover,*
$$\lim_{t \to \infty} Pr\{a_t = a_{max}\} = 1.$$

Proposition 3 characterizes equilibrium dynamics after a project is funded. In any profit-

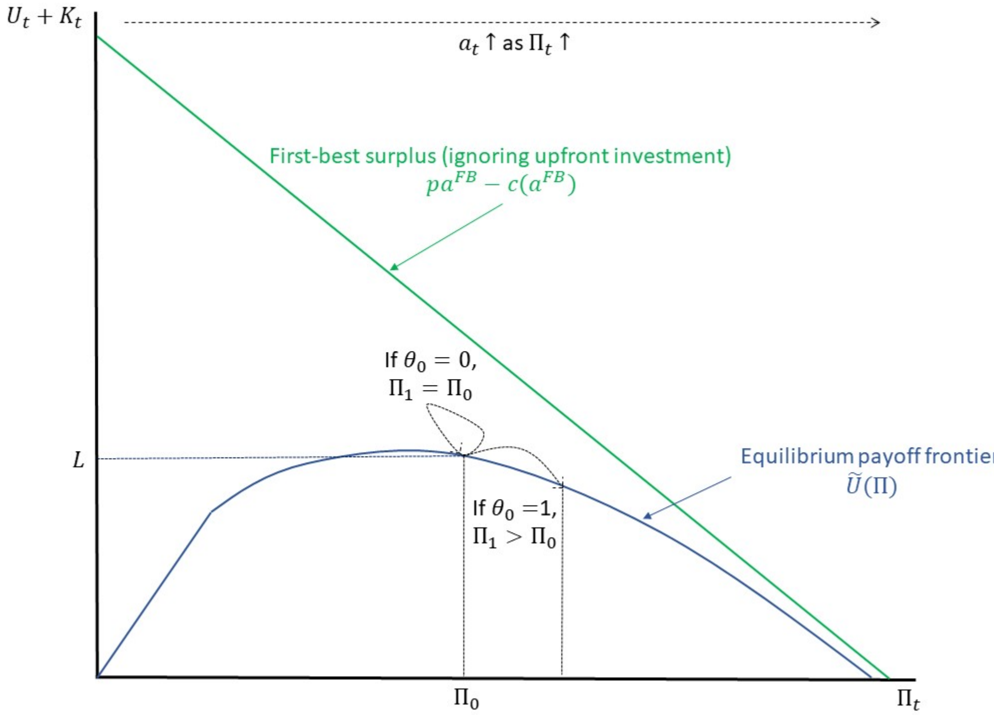

Figure 3: An illustration of the equilibirum payoff frontier. Note that the y-axis is the *sum* of the worker's and creditor's payoffs, which by Lemma 1, is sufficient to identify the manager's equilibrium payoff.

maximizing equilibrium, neither the worker nor the creditor earns rent at the start of the game: $U_0 = 0$ and $K(0, \Pi_0) = L$. From this starting point, effort increases stochastically over time: if $\theta_t = 0$, then effort remains constant, while if $\theta_t = 1$, it strictly increases. This process continues until effort reaches the steady-state level a_{max}, by which point the creditor has been fully repaid. See Figure 3 for an illustration.

To see the intuition for this result, suppose that $a_{max} < a^{FB}$ and define the *game without debt* by setting $L = 0$. In the game without debt, we can ignore the creditor entirely, set $r_t = 0$ in all t, and focus on the two-person game between the manager and the worker. The equilibrium payoff frontier of this game is $\tilde{U}(\Pi)$, where Lemma 1 says that $K(U, \Pi) = \tilde{U}(\Pi) - U$. We can therefore characterize profit-maximizing equilibria by setting the manager's initial profit, Π_0, so that $\tilde{U}(\Pi_0) = L$ in the game without debt.

If $\tilde{U}(\Pi) > 0$, then (IC) is slack in the game without debt. Total surplus, $\tilde{U}(\Pi) + \Pi$, is therefore increasing in Π_0, since decreasing the worker's payoff can always be accomplished by either increasing effort, which increases total surplus, or decreasing wages, which leaves total surplus unaffected. This fact implies that if $L > 0$, then $w_t + r_t = y_t$ whenever $\tilde{U}(\Pi_H) > 0$, since increasing $w_t + r_t$ and decreasing $U_{t+1} + K_{t+1}$ by the same amount increases total surplus. This argument pins down equilibrium payoffs and dynamics following $\theta = 1$. Since Π_L does not enter (DE), it can be freely adjusted. Consequently, the concavity of $\tilde{U}(\cdot)$ implies that $\Pi_L = \Pi$ is optimal. We conclude that following high output, $w + r = y$ and effort increases in the next period, while following low output, effort remains the same in the next period. These dynamics continue until all obligations are repaid, $\tilde{U}(\Pi_H) = 0$, after which $a_t = a_{max}$ in every period.

Proposition 3 implies that debt decreases effort and results in productivity dynamics: unless L (and hence $K(0, \Pi_0)$) is small, effort slowly and stochastically increases to the steady state a_{max}. The fact that debt decreases effort and productivity is consistent with our empirical results.

We now turn to compensation dynamics. While $K(U_t, \Pi_t) + U_t$ is pinned down in every profit-maximizing equilibrium, $K(U_t, \Pi_t)$ and U_t are not. Different combinations of these payoffs correspond to different equilibrium wage and repayment paths. Our next result identifies two extreme paths that repay the creditor either as quickly or as slowly as possible. In both of these paths, wages are (at least weakly) backloaded and repayments are frontloaded.[20]

Corollary 1 *Each of the following is part of a profit-maximizing equilibrium.*

1. The fastest repayment equilibrium: $r_t = y_t$ *whenever* $K(U_H, \Pi_H) > 0$. *Once* $K(U, \Pi) = 0$, $w_t = y_t$ *whenever* $U_H > 0$.

2. The slowest repayment equilibrium: *whenever* $K(0, \Pi_H) > 0$, $r_t = y_t - \frac{c(a_t)}{p}$. *In every* $t \geq 0$, $w_t = \frac{c(a_t)}{p}$ *and* $U_t = 0$.

[20] Small changes to the modeling environment would break the manager's indifference across these payment paths. For instance, if the creditor was more impatient than the worker, or if it incurred higher opportunity costs from not being repaid, then it would be optimal to repay the creditor as quickly as possible. If the worker was more impatient, in contrast, then it would be optimal to repay the creditor as slowly as possible.

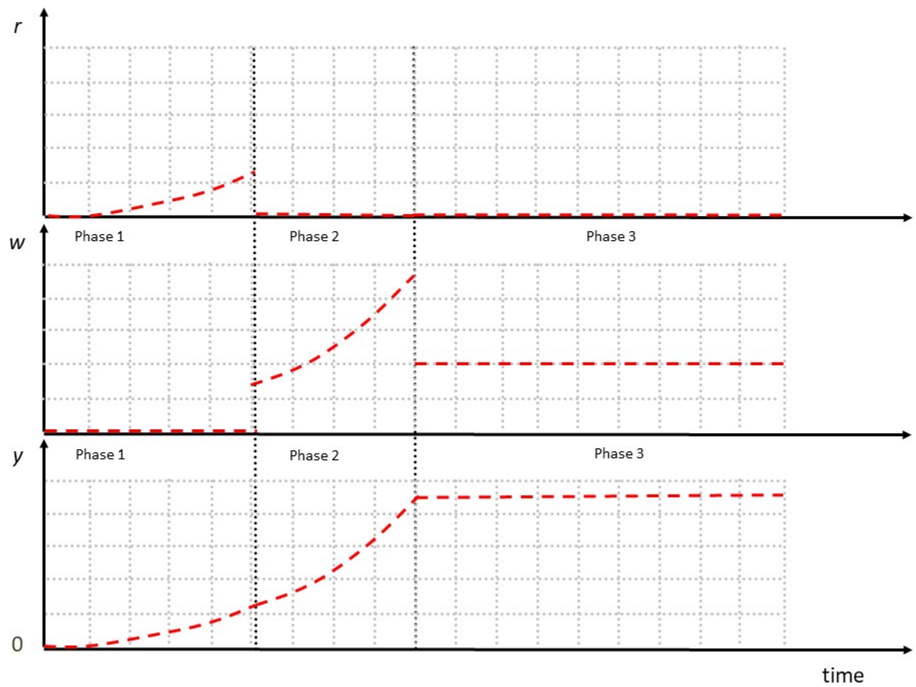

Figure 4: Dynamics in the fastest repayment equilibrium for $p = 1$.

Corollary 1 follows from Lemma 1 and the proof of Proposition 3. Before the worker's effort reaches a_{max}, every profit-maximizing equilibrium entails the same *total* payment, $w_t + r_t$; the only question is how much of this total payment goes to the creditor. To repay the creditor as quickly as possible, the manager fully backloads wages until after the creditor has been repaid. In the slowest repayment path, by contrast, she exactly compensates the worker for his cost in each period.

This result implies that productivity dynamics can persist even after the creditor has been repaid. In particular, if $K(U, \Pi) = 0$ but $U > 0$ and $U_H > 0$, as can be the case in the fastest repayment equilibrium, then effort temporarily remains depressed even after the manager has repaid her debts. In Section 5, we will show that manager-worker collusion can induce fast repayment and so lead to these kinds of lingering dynamics. This result resonates with Table 3, which shows that even conditional on current leverage changes, *past* leverage is negatively correlated with current productivity. For $p = 1$, Figure 4 presents effort, wage, and repayment dynamics in the fastest repayment equilibrium.

In this model, dynamics arise from a fundamental asymmetry between the creditor and the worker. The worker's incentives depend on his *future* wages, while the creditor does not care about the timing of repayments. Consequently, wages are optimally back-loaded, while debt repayments are front-loaded to maximize Π_H and so relax (DE) as much as possible.

5 Collusion Between the Manager and the Worker

This section explores the possibility of manager-worker collusion. We show that the creditor optimally deters collusion by threatening to liquidate the project unless it is repaid promptly. An unlucky firm might then be inefficiently liquidated, further increasing the equilibrium cost of debt.

Introducing collusion to our model entails both conceptual and technical challenges. The technical challenge is that, under a reasonable model of manager-worker collusion, Lemma 1 does not hold, so we have to separately track the creditor's and the worker's continuation payoffs. The conceptual challenge is that, given the manager's lack of commitment, manager-worker collusion should be self-enforcing in the context of their ongoing relationship. That is, collusive outcomes should be equilibria.

Given these challenges, we focus on a simple binary-effort version of the model: $a_t \in \{0, y\}$, where (with abuse of notation) $c(0) = 0$ and $c(y) = c$. Since this simplification limits equilibrium effort dynamics, we focus on wage and liquidation dynamics. In Online Appendix E, we show that similar takeaways hold in a setting with continuous effort. In particular, the manager optimally backloads worker pay whenever collusion is a binding concern, leading to similar implications for the relationship between debt, pay, and productivity.

To understand the way we approach manager-worker collusion, consider the following thought experiment. Suppose that after the creditor accepts the formal contract, the manager and the worker can agree on *any* continuation equilibrium. Then, they will never choose an equilibrium in which the worker punishes the manager for reneging on r_t, since they could instead split r_t in a way that benefits both of them. Consequently, the creditor cannot rely on the worker to punish deviations in r_t; it can guarantee repayment only by threatening liquidation.

Online Appendix C formalizes this thought experiment. This appendix shows that an equilibrium is immune to collusion if, whenever output is produced, the worker and the manager would jointly prefer to pay r_t rather than pocketing it and acting as if no output had been produced. We say that an equilibrium satisfying this (sufficient) condition is a truth-telling equilibrium.

Definition 1 *An SPE σ^* is a **truth-telling equilibrium** if, at every on-path history h^t immediately before θ_t is realized,*

$$E_{\sigma^*}\left[-(1-\delta)r_t+\delta\left(\Pi_{t+1}+U_{t+1}\right)|h^t,\theta_t=1\right]\geq E_{\sigma^*}\left[\delta\left(\Pi_{t+1}+U_{t+1}\right)|h^t,\theta_t=0\right]. \tag{7}$$

Online Appendix C discusses truth-telling equilibria in more detail. One attractive feature of (7) is that it resembles the truth-telling constraints imposed in capital dynamics models without a worker, such as Clementi and Hopenhayn (2006) and DeMarzo and Fishman (2007). Relative to those papers, the key difference is that (7) depends on the *sum* of the worker's and the manager's payoffs rather than the manager's payoff alone, reflecting the fact that collusion entails a joint rather than individual deviation. This model also relates to the static analyses of collusion in Tirole (1986) and Biais and Gollier (1997).

We can write (7) recursively as the following **truth-telling** constraint:

$$(1-\delta)r\leq\delta(\Pi_H+U_H-\Pi_L-U_L). \tag{TT}$$

Let $K^T(U,\Pi)$ be the solution to (5) subject to (PK-A)-(CE) and (TT), given $a_t\in\{0,y\}$. In a profit-maximizing truth-telling equilibrium, on-path continuation payoffs always lie on $K^T(\cdot)$. Our next result partially characterizes this equilibrium payoff frontier, focusing on the relationship between debt, pay, and liquidation. The proof is in Appendix A.

Proposition 4 *Suppose*

$$\frac{c}{p} \leq \frac{\delta}{1-\delta}(py - c). \tag{8}$$

Then,

1. *Fix* $\Pi \geq \frac{p(1-\delta)}{1-(1-p)\delta}y$. *If* $U \in \mathbb{R}_+$ *is such that* $K^T(U,\Pi) > 0$, *then for any* $U' > U$ *such that* (U',Π) *can be attained in a truth-telling equilibrium,* $K^T(U,\Pi) + U < K^T(U',\Pi) + U'$.

2. *If* $K^T(U_H,\Pi_H) > 0$, $w = 0$ *and* $r = y$.

3. *If* $K^T(U,\Pi) > 0$, *then liquidation occurs with strictly positive probability in the continuation equilibrium.*

Condition (8) ensures that if $L = 0$, then there exists an equilibrium with $a_t = y$ in every $t \geq 0$. The first part of Proposition 4 says that Lemma 1 does not hold for truth-telling equilibria. The reason is that, unlike wages, repayments must satisfy (TT). This asymmetry creates an additional reason to frontload repayments, since doing so allows the manager to pay off her debt and so "escape" the truth-telling constraint as quickly as possible. Consequently, the second part of Proposition 4 says that the manager initially uses her entire output to repay the creditor. While the creditor is being repaid, the worker receives no wages and is motivated only by the promise of future compensation. The final part of the result identifies the cost of deterring collusion: so long as debt remains, liquidation occurs with positive probability on the equilibrium path. In short, the creditor threatens liquidation to deter collusion. The manager minimizes the probability of liquidation by frontloading loan repayment and backloading worker compensation. As a result, total surplus and the worker's continuation payoff depend on current and past debts.

Proposition 4 says that *every* profit-maximizing truth-telling equilibrium entails deferred worker compensation, which strengthens Corollary 1's result that *there exist* profit-maximizing equilibria with deferred compensation. Since the project is sometimes liquidated in equilibrium, profit-maximizing equilibria eventually converge to one of two steady states:

either the loan is repaid and continuation surplus converges to $\frac{\delta}{1-\delta}(py - c)$, or the firm is liquidated for failing to repay its debts.

Truth-telling equilibria highlight an additional commitment problem faced by firms, which is that members of the firm might *jointly* deviate when it is in their mutual interest. Ali and Liu (2018) explores the constraints imposed by this type of "coalitional deviation." Since deterring collusion entails the risk of inefficient liquidation, firms have the incentive to develop institutions that make collusion less feasible. A strong whistleblower culture, which encourages workers to report misbehavior by their managers, might serve as one such institution. As we discuss in the next section, a firm's capital structure can also affect the feasibility of collusion.

6 Discussion

This section informally discusses several extensions and concludes.

The role of corporate governance: While the manager-creditor contract is easiest to interpret as debt, our mechanism also applies to other financial contracts, such as external equity. We briefly speculate on the similarities and differences between debt and external equity.

So long as output is non-contractible, debt and external equity have similar negative effects on morale. The reason is that equity, like debt, suffers from a commitment problem, since the manager can always divert profits rather than paying promised dividends. Consequently, promises to equity holders must be made credible just like debt repayments.

One difference between debt and equity is that equity contracts do not include the threat of liquidation. However, equity *can* entail voting rights; if it does, then equity holders can fire misbehaving managers. In principle, the threat of firing can be used to replicate the role of liquidation in the optimal contract.

A more fundamental point is that equity affects who has control rights over the firm, which raises questions about the role of corporate governance in relational contracts. On one hand, external control might facilitate relational contracts, because equity holders can limit managerial discretion. For instance, by issuing equity to both creditors and workers, a manager can commit to either pay *both* of them or *neither* of them. Tying compensation to repayments in this way can deter manager-worker collusion by making it harder for the manager to "secretly" pay the worker. Conversely, external control can create new commitment problems, as equity holders might be tempted to renege on the firm's relational obligations. We hope that future work explores how corporate governance affects relational contracts.

Saving and borrowing: Apart from the initial loan, the manager neither borrows nor saves in our model. If the manager could borrow additional funds from the creditor, then she could pay the worker more, which would decrease the worker's promised continuation utility at the cost of increasing the creditor's promised continuation utility. Without collusion, Lemma 1 suggests the *sum* of these utilities determines productivity, so the manager would not benefit from further borrowing. With collusion, Proposition 4 suggests that further borrowing would increase the probability of liquidation and would, therefore, be strictly inefficient. Thus, the manager would not benefit from further borrowing.

The effects of savings are a little more complicated. If the manager can still access her savings after deviating, then we believe that not much would change; she would still repay her obligations as quickly as possible and so would not save in profit-maximizing equilibria. In contrast, if the creditor or worker could stop the manager from accessing savings following a deviation, then accumulated savings could serve as a bond to guarantee the manager's promises. Such bonds mitigate commitment problems and could alter equilibrium dynamics.

Other types of investments: In some settings, the manager can choose the scale of her investment, with larger-scale investments having higher up-front costs and higher returns. Our model suggests two opposing equilibrium distortions in such settings. On one hand, debt decreases effort, which should push the manager to pursue smaller, less expensive projects.

On the other hand, investment can sometimes ease commitment problems, because larger projects might imply that the manager has more to lose following a deviation (Klein and Leffler (1981); Halac (2015); Englmaier and Fahn (2019)). We can construct examples in which either of these two forces dominate, leading to either "insufficient" or "excessive" equilibrium investment relative to first-best.

Our results suggest that firms might also benefit from altering their investments in other ways. In the absence of manager-worker collusion, profits are higher when the worker can more severely punish the manager. To give workers access to more severe punishments, the firm might therefore distort its investments towards projects that are highly reliant on effort. Such labor-intensive investments might facilitate relational contracts better than more capital-intensive investments.[21]

Conclusion: Debt exacerbates the commitment problems that live at the heart of manager-worker relationships. Our analysis exposes how these commitment problems influence worker morale, and in doing so, shape productivity dynamics. We show that debt negatively affects both effort and wages, which resonates with our stylized facts relating higher leverage to lower employee morale, lower compensation, and lower productivity. More generally, our analysis illustrates how a firm's organizational structure can significantly impact its interactions with capital markets. By determining how firms respond to investment opportunities, these spillovers influence productivity, profitability, and growth.

[21] We thank an anonymous referee for emphasizing this point.

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A Omitted Proofs

A.1 Proof of Proposition 1

In the commitment game, suppose the manager offers a contract with $r_t(y_t) = \frac{L}{p}1\{y_t \geq \frac{L}{p}\}$, $w_t(y_t) = \frac{c(a^{FB})}{p}1\{y_t = a^{FB}\}$, and $l_t = 0$ in each $t \geq 0$. The creditor earns $pr_t(y_t) = L$ from this contract, while the worker earns $p\frac{c(a^{FB})}{p} - c(a^{FB}) = 0$ from choosing a^{FB} and no more than 0 from choosing any other effort. Therefore, this contract induces the creditor to fund the project and the worker to choose $a_t = a^{FB}$ in each period. The manager earns $pa^{FB} - c(a^{FB}) - L > 0$, which is the maximum attainable surplus in any equilibrium. Therefore, this contract is profit-maximizing, and any profit-maximizing contract must fund the project and induce a^{FB} in each period. ■

A.2 Proof of Proposition 2

Consider the following strategy in the bilateral game: the manager offers the creditor a contract with $l_t = 0$ in each t, which is accepted. The worker chooses $a_t = a^{FB}$. Whenever $\theta_t = 1$ on the equilibrium path, $r_t = \frac{L}{p}$ and $w_t = \frac{c(a^{FB})}{p}$. Whenever $\theta_t = 0$, $r_t = w_t = 0$. Following any deviation, $r_t = a_t = w_t = 0$.

Under the strategy, the manager earns $pa^{FB} - c(a^{FB}) - L > 0$. She is willing to pay $r_t = \frac{L}{p}$ as long as

$$(1-\delta)\frac{L}{p} \leq \delta(pa^{FB} - c(a^{FB}) - L)$$

which holds for $L \leq \frac{\delta p}{1-\delta+\delta p}(pa^{FB} - c(a^{FB}))$. Payments are feasible because $r_t + w_t = \frac{L}{p} + \frac{c(a^{FB})}{p} < a^{FB}$ is implied by $L < pa^{FB} - c(a^{FB})$. The manager earns $pa^{FB} - c(a^{FB}) - L > 0$ from this equilibrium, which is the maximum possible total surplus. Thus, if $L \leq \frac{\delta p}{1-\delta+\delta p}(pa^{FB} - c(a^{FB}))$, any profit-maximizing equilibrium entails $a_t = a^{FB}$ in each period t on the equilibrium path.

Now, in any equilibrium, (DE) requires that in any $t \geq 0$ such that $\theta_t = 1$,

$$(1-\delta) r_t \leq \delta \left(p a^{FB} - c(a^{FB}) - E[K_{t+1} | h^t] \right). \tag{9}$$

For each $t \geq 0$, define $\mathcal{H}(t) \equiv \{h^t | \theta_t = 1, \theta_{t'} = 0 \; \forall \; t' < t\}$ as the set of histories such that $\theta_t = 1$ for the first time in period t. The project is funded only if

$$\sum_{t=0}^{\infty} \delta^t (1-p)^t p E \left[(1-\delta) r_t + \delta K_{t+1} | \mathcal{H}(t) \right] \geq L.$$

Applying (9) to this expression yields

$$\delta p \sum_{t=0}^{\infty} \delta^t (1-p)^t (p a^{FB} - c(a^{FB})) \geq L$$

or $\frac{\delta p}{1-\delta+\delta p}(p a^{FB} - c(a^{FB})) \geq L$. We conclude that the project is funded only if this inequality holds. ■

A.3 Proof of Lemma 1

We first prove that $K(\tilde{U}(\Pi), \Pi) = 0$ for any $(U, \Pi) \in E$. Towards contradiction, suppose $K(\tilde{U}(\Pi), \Pi) > 0$ for some $(U, \Pi) \in E$. Then, there exists some future period in which $r > 0$. In this period, consider the following perturbation: decrease r to $\tilde{r} = r - \epsilon$ and increase w to $\tilde{w} = w + \epsilon$. For sufficiently small $\epsilon > 0$, these perturbed payoffs continue to satisfy (DE) and (LL) in that and all previous periods because $r > 0$ and $r + w = \tilde{r} + \tilde{w}$, while (IC) is relaxed in that and all previous periods. However, then $U < \tilde{U}(\Pi)$ and we obtain a contradiction. Therefore, $K(\tilde{U}(\Pi), \Pi) = 0$ for all $(U, \Pi) \in E$.

Next, we prove that $K(U, \Pi) = \tilde{U}(\Pi) - U$. If $U = \tilde{U}(\Pi)$, then this result holds by the previous argument. Suppose $0 \leq U < \tilde{U}(\Pi)$. Then, we claim that $K(U, \Pi) > 0$. To prove this, consider the equilibrium in which the worker and manager earn $\tilde{U}(\Pi)$ and Π, respectively. We claim that there exists some period t and history h^t at the start of that period such that (i) $w_t > 0$ if demand is high in that period, (ii) $E[U_t|h^t] > 0$, and (iii) $E[U_{t'}|h^{t'}] > 0$ for any $h^{t'}$ that precedes h^t.

To prove this, for each $\tau \geq 1$, define $\mathcal{H}^\tau$ as the set of histories such that $E[U|h^\tau] = 0$, but $E[U|h^t] > 0$ for all h^t that precedes h^τ. Define $\mathcal{H}^\infty$, with element $h^\infty \in \mathcal{H}^\infty$, as the set of infinite-horizon histories for which $E[U|h^\tau] > 0$ for every h^τthat precedes h^∞. Then, $\mathcal{H}^\tau \cap \mathcal{H}^{\tau'} = \emptyset$ and $\bigcup_{\tau=0}^{\infty} \mathcal{H}^\tau = \mathcal{H}$,[22] so the worker's payoff can be written as follows:

$$U = \sum_{\tau=0}^{\infty} \Pr_{\sigma^*}\{\mathcal{H}^\tau\} \left(E\left[(1-\delta) \sum_{t=0}^{\tau-1} \delta^t (w_t - c(a_t)) | \mathcal{H}^\tau \right] \right).$$

Since $U > 0$ and $c(a_t) \geq 0$, it cannot be that $w_t \equiv 0$ in this expression. That is, there must exist some τ, $h^\tau \in \mathcal{H}^\tau$, and h^t that precedes h^τ such that $w_t > 0$ with positive probability at h^t. By definition, $E[U_t|h^t] > 0$ and $E[U_{t'}|h^{t'}] > 0$ for any $h^{t'}$ that precedes h^t.

Consider decreasing $w_t > 0$ at this h^t and increasing r_t by the same amount. This perturbation satisfies (DE) and (LL) because $w_t > 0$ and $w_t + r_t$ is constant, while (IC) is slack at h^t and every predecessor history, so it continues to hold for a sufficiently small perturbation. Since $K + U$ remains constant in this perturbation, which can be performed for any $U > 0$, $K(U, \Pi) > 0$ whenever $0 < U < \tilde{U}(\Pi)$. If $U = 0$, then $K(U, \Pi) > 0$, and we can decrease r in some period holding $w + r$ fixed to transfer utility from the creditor to the worker. Therefore, $K(U, \Pi) + U$ is constant in U, and $K(U, \Pi) = \tilde{U}(\Pi) - U$. ■

A.4 Proof of Proposition 3

Lemma 1 says that we can characterize $K(U, \Pi)$ using $\tilde{U}(\cdot)$. Define Π_{max} as the maximum Π such that there exists a U with $(U, \Pi) \in E$. Let $S^{FB} = pa^{FB} - c(a^{FB})$. To characterize $\tilde{U}(\cdot)$, it suffices to consider equilibria with $L = 0$ and $r_t = 0$ in every $t \geq 0$.

We proceed in four steps. First, we show that $\tilde{U}(\cdot)$ has the shape shown in Figure 3: it is concave, with $\tilde{U}(\Pi) + \Pi$ increasing in Π and strictly so unless $\tilde{U}(\Pi) + \Pi = S^{FB}$. Second, we

[22]Note that this union, and the summation below, both include $\mathcal{H}^\infty$.

prove Statement 2 of Proposition 3, which characterizes how the manager's payoff evolves over time in response to shocks. Next, we prove Statement 1 of Proposition 3. Finally, we show that these two statements imply Statement 3 of Proposition 3.

Part 1: $\tilde{U}(\cdot)$ is concave, with $\tilde{U}(\Pi) + \Pi$ increasing in Π and strictly so unless $\tilde{U}(\Pi) + \Pi = S^{FB}$

Since the public randomization device can be used to randomize between continuation equilibria, $\tilde{U}(\Pi)$ is concave.

For any Π such that $\tilde{U}(\Pi) > 0$, we show that we can increase Π and decrease $\tilde{U}(\Pi)$ in a way that increases total surplus. Since $\tilde{U}(\Pi) > 0$, as in the proof of Lemma 1, there exists some history h^t such that $w_t > 0$, $\mathbb{E}[U_t|h^t] > 0$, and $E[U_{t'}|h^{t'}] > 0$ in every $h^{t'}$ that precedes h^t. Consider decreasing w_t at this history. Doing so relaxes both (DE) and (LL), while (IC) continues to hold in all periods because it was previously slack. This perturbed strategy is therefore an equilibrium with higher Π, lower U, and the same total surplus. We conclude that $\tilde{U}(\Pi) + \Pi$ is weakly increasing in Π.

The concavity of $\tilde{U}(\cdot)$ implies that the slope of $\tilde{U}(\Pi)+\Pi$ is decreasing in Π. Consequently, there exists $\bar{\Pi} \in [0, \Pi_{max}]$ such that $\tilde{U}(\Pi) + \Pi$ is strictly increasing for $\Pi < \bar{\Pi}$ and weakly increasing for $\Pi \geq \bar{\Pi}$.

We now show that $\tilde{U}(\Pi) + \Pi$ is strictly increasing unless $\tilde{U}(\Pi) + \Pi = S^{FB}$. Since $\tilde{U}(\Pi) + \Pi \leq S^{FB}$, it suffices to show that if $\bar{\Pi} < \Pi_{max}$, then $\tilde{U}(\bar{\Pi}) + \bar{\Pi} = S^{FB}$. Towards contradiction, suppose that $\bar{\Pi} < \Pi_{max}$ but $\tilde{U}(\bar{\Pi}) + \bar{\Pi} < S^{FB}$. Denote $\tilde{U}(\bar{\Pi}) + \bar{\Pi} = \bar{S}$, and note that $\tilde{U}(\Pi) + \Pi = \bar{S}$ for any $\Pi \in [\bar{\Pi}, \Pi_{max}]$.

We first argue that if $\Pi \geq \bar{\Pi}$, then $\Pi_L, \Pi_H \geq \bar{\Pi}$. If $\Pi_H < \bar{\Pi}$, then we could perturb the equilibrium by increasing Π_H. This perturbation relaxes (DE); it does not violate (IC), because $\bar{\Pi} < \Pi_{max}$ implies $\tilde{U}(\bar{\Pi}) > 0$; and no other constraints are affected. Consequently, this perturbation is also an equilibrium; since $\tilde{U}(\Pi_H)+\Pi_H$ is strictly increasing for $\Pi_H < \bar{\Pi}$, total surplus in this perturbed equilibrium strictly exceeds $\bar{S}$, which contradicts the definition of $\bar{S}$. A similar argument applies if $\Pi_L < \bar{\Pi}$, so we conclude that if $\Pi \geq \bar{\Pi}$, then $\Pi_L, \Pi_H \geq \bar{\Pi}$.

Now, define $\bar{a}$ as the equilibrium effort in $t = 0$ of a profit-maximizing equilibrium with payoffs $\Pi_0 = \bar{\Pi}$, $U_0 = \tilde{U}(\bar{\Pi})$. Then,

$$\begin{aligned}\tilde{U}(\bar{\Pi}) + \bar{\Pi} &= (1-\delta)(p\bar{a} - c(\bar{a})) + \delta\left(p\left(\tilde{U}(\Pi_H) + \Pi_H\right) + (1-p)\left(\tilde{U}(\Pi_L) + \Pi_L\right)\right) \\ &= (1-\delta)(p\bar{a} - c(\bar{a})) + \delta\bar{S},\end{aligned}$$

or $\bar{S} = \tilde{U}(\bar{\Pi}) + \bar{\Pi} = p\bar{a} - c(\bar{a})$. Thus, total surplus $\bar{S}$ can be attained in a stationary equilibrium with effort $\bar{a} < a^{FB}$.

Since $\tilde{U}(\Pi_{max}) = 0$, we know that $\Pi_{max} = p\bar{a} - c(\bar{a})$. Consider the equilibrium that gives the firm and worker Π_{max} and $\tilde{U}(\Pi_{max}) = 0$, respectively. Because total surplus is increasing in Π, there exists such an equilibrium in which $a = \bar{a}$, $w = \frac{c(\bar{a})}{p}$, and $\Pi_L = \Pi_H = \Pi_{max}$. This equilibrium is stationary, so (DE) becomes

$$\frac{c(\bar{a})}{p} \leq \frac{\delta}{1-\delta}\left(\bar{a} - c(\bar{a})\right).$$

Then, $\bar{a}$ will be the most efficient effort that satisfies this constraint, which by (6) implies $\bar{a} = a_{max} < a^{FB}$.

Suppose that $\bar{\Pi} < \Pi_{max}$, so $\tilde{U}(\bar{\Pi}) > 0$. For $\Pi = \bar{\Pi}$, we have shown that $\Pi_L, \Pi_H \geq \bar{\Pi}$, which means that the worker's continuation payoff is no more than $\tilde{U}(\bar{\Pi})$. Therefore, if w is the on-path wage paid in the corresponding equilibrium, then

$$\tilde{U}(\bar{\Pi}) \leq (1-\delta)(wp - c(a_{max})) + \delta\tilde{U}(\bar{\Pi}).$$

Consequently, $\tilde{U}(\bar{\Pi}) \leq wp - c(a_{max})$, so $\tilde{U}(\bar{\Pi}) > 0$ implies $w > \frac{c(a_{max})}{p}$. But (DE) holds with equality with $w = \frac{c(a_{max})}{p}$ and $\Pi_H = \Pi_{max}$, which means it is violated if $w > \frac{c(a_{max})}{p}$. We conclude that $\bar{\Pi} = \Pi_{max}$, as desired.

Part 2: Statement 2 of Proposition 3

First, we show that $\Pi_L = \Pi$ **whenever** $\Pi < \bar{\Pi}$**.** Denote the right-hand and left-hand derivatives of $\tilde{U}$ by $\partial_+\tilde{U}$ and $\partial_-\tilde{U}$, respectively. Note that both half-derivatives exist because $\tilde{U}(\cdot)$ is concave.

The first step is to show that for any $\Pi \in (0, \bar{\Pi})$, $\partial_+\tilde{U}(\Pi_L) \leq \partial_+\tilde{U}(\Pi)$ and $\partial_-\tilde{U}(\Pi) \leq \partial_-\tilde{U}(\Pi_L)$. Suppose that $\partial_+\tilde{U}(\Pi_L) > \partial_+\tilde{U}(\Pi)$, which, since $\tilde{U}$ is concave, implies $\Pi_L < \Pi$. Since $\Pi < \Pi_{max}$, $\tilde{U}(\Pi) > 0$. Thus, play remains an equilibrium if we slightly increase Π_L. Doing so increases Π at rate $(1-p)\delta$ and decreases U at rate $(1-p)\delta\partial_+\tilde{U}(\Pi_L)$. This slope of this perturbation is therefore $\partial_+\tilde{U}(\Pi_L)$; since this perturbation is feasible, this slope bounds $\partial_+\tilde{U}(\Pi)$ from below. Thus, $\partial_+\tilde{U}(\Pi_L) \leq \partial_+\tilde{U}(\Pi)$. If $\partial_-\tilde{U}(\Pi_L) < \partial_-\tilde{U}(\Pi)$, then we can make a similar argument by decreasing Π_L. This proves that $\partial_+\tilde{U}(\Pi_L) \leq \partial_+\tilde{U}(\Pi)$ and $\partial_-\tilde{U}(\Pi) \leq \partial_-\tilde{U}(\Pi_L)$.

The second step is to argue that $\partial_+\tilde{U}(\Pi)$ is strictly decreasing for any $\Pi < \bar{\Pi}$. It is weakly decreasing because $\tilde{U}(\cdot)$ is concave; thus, it suffices to show that $\tilde{U}(\cdot)$ is never linear.

Towards contradiction, suppose that $\tilde{U}(\cdot)$ is linear on the interval $\Pi^A < \Pi^B$. For $i \in \{A, B\}$, let $(w^i, a^i, \Pi_L^i, \Pi_H^i)$ be the wage, effort, and continuation payoffs for the equilibrium associated with manager payoff Π^i. Then, for any $\alpha \in (0,1)$, define $w = \alpha w^A + (1-\alpha)w^B$, with a, Π_L, and Π_H as similar convex combinations of the corresponding variables. These convex combinations also form an equilibrium. If $a^A \neq a^B$, then the worker's payoff from this convex combination strictly exceeds $\alpha\tilde{U}(\Pi^A) + (1-\alpha)\tilde{U}(\Pi^B)$ because $c(\cdot)$ is strictly convex. This would contradict that $\tilde{U}(\cdot)$ is linear on this region, so it must be that $a^A = a^B$.

It must be that $\Pi_L^A \geq \Pi^A$, since otherwise $\partial_+\tilde{U}(\Pi_L^A) > \partial_+\tilde{U}(\Pi^A)$, contradicting our previous proof that $\partial_+\tilde{U}(\Pi_L) \leq \partial_+\tilde{U}(\Pi)$. It must also be that $\Pi_L^A \leq \Pi^A$, since otherwise we could slightly decrease Π_L^A and extend the linear segment $[\Pi^A, \Pi^B]$ of $\tilde{U}(\cdot)$. Thus, $\Pi_L^A = \Pi^A$. A similar argument implies $\Pi_L^B = \Pi^B$.

Since $\Pi < \bar{\Pi}$, $a^A = a^B < a^{FB}$.[23] Therefore, (DE) must bind, since otherwise it would be

[23] To see this point, note first that $w = a$; otherwise, we could increase U by increasing w while holding

feasible to increase a and w at rate 1, increasing total surplus while holding the manager's payoff fixed. Binding (DE) implies

$$\Pi^A = (1-\delta)pa^A + \delta(1-p)\Pi^A_L,$$

which, together with $\Pi_A = \Pi^L_A$, implies

$$\Pi^A = \frac{1-\delta}{1-\delta(1-p)}a^A.$$

Similarly,

$$\Pi^B = \frac{1-\delta}{1-\delta(1-p)}a^B.$$

We conclude that $\Pi^A = \Pi^B$, since $a^A = a^B$. This contradicts $\Pi^A < \Pi^B$, so we conclude that $\partial_+\tilde{U}(\Pi)$ is strictly decreasing on $\Pi < \bar{\Pi}$. This completes the second step.

The final step is to note that $\partial_+\tilde{U}(\Pi) \leq \partial_-\tilde{U}(\Pi)$, because $\tilde{U}(\cdot)$ is concave. We have therefore shown that

$$\partial_+\tilde{U}(\Pi_L) \leq \partial_+\tilde{U}(\Pi) \leq \partial_-\tilde{U}(\Pi) \leq \partial_-\tilde{U}(\Pi_L). \tag{10}$$

We have shown that $\partial_+\tilde{U}(\cdot)$ is strictly decreasing, and a similar argument shows that $\partial_-\tilde{U}(\cdot)$ is strictly decreasing. Thus, (10) implies $\Pi_L = \Pi$.

Second, we show that $\Pi_H > \Pi$ whenever $\Pi < \bar{\Pi}$. Towards contradiction, suppose $\Pi_H \leq \Pi < \bar{\Pi}$. The upper bound of (LL) must bind, since otherwise, we could increase Π_H and w so that $(1-\delta)w + \delta\Pi_H$ is constant. This perturbation holds the manager's payoff fixed

total surplus fixed, which would contradict $\Pi < \bar{\Pi}$. Now, suppose $a > a^{FB}$. In that case, we could decrease a and w at the same rate, increasing total surplus holding the manager's payoff constant and thereby contradicting the definition of $\tilde{U}(\cdot)$. Thus, if $a \geq a^{FB}$, then $a = a^{FB}$, which can hold only if $a_{max} = a^{FB}$ and so $\tilde{U}(\bar{\Pi}) + \bar{\Pi} = pa^{FB} - c(a^{FB})$. Suppose $a^A = a^{FB}$. If $\Pi^A_H \geq \bar{\Pi}$, then $\Pi^A_L \geq \Pi^A$ implies $\tilde{U}(\Pi^A) + \Pi^A = pa^{FB} - c(a^{FB})$, contradicting $\Pi^A < \bar{\Pi}$. But if $\Pi^A_H < \bar{\Pi}$, then slightly decreasing a entails a second-order loss in total surplus, while slightly increasing Π^A_H entails a first-order gain. We could therefore decrease a and increase Π^A_H at the appropriate rates to increase total surplus holding the manager's payoff constant. We conclude that if $\Pi < \bar{\Pi}$, then $a = a^{FB}$.

while increasing total surplus, since $\tilde{U}(\Pi_H) + \Pi_H$ is strictly increasing. Thus, the original equilibrium could not have been optimal.

This argument implies that $w = a$ whenever $\Pi_H < \bar{\Pi}$. Consequently,

$$\Pi = \delta p \Pi_H + \delta(1-p)\Pi_L \leq \delta \Pi,$$

where the inequality follows because $\Pi_H \leq \Pi$ and $\Pi_L = \Pi$. This is a contradiction because $\delta < 1$, so $\Pi_H > \Pi$.

Part 3: Statement 1 of Proposition 3

We argue that $a = a^*(\Pi)$ in the equilibrium yielding payoffs $(\tilde{U}(\Pi), \Pi))$. If $\Pi \geq \bar{\Pi}$, then $a = a^*(\Pi) = a_{max}$ by the argument in Part 1. If $\Pi < \bar{\Pi}$, then we have already argued in Part 2 that $a < a^{FB}$ and (DE) binds. Therefore,

$$\Pi = (1-\delta)pa + \delta(1-p)\Pi_L.$$

Since $\Pi_L = \Pi$, we conclude that

$$a = \frac{1-\delta(1-p)}{(1-\delta)p}\Pi = a^*(\Pi),$$

as desired.

Part 4: Statement 3 of Proposition 3

Since $\Pi_L = \Pi$ and $\theta = 1$ with probability $p > 0$, it suffices to show that with probability 1, any profit-maximizing equilibrium reaches a period in which $\Pi_H \geq \bar{\Pi}$. So long as $\Pi_H < \bar{\Pi}$, (LL) binds, which immediately implies that $w_t + r_t = a_t$ whenever $\theta_t = 1$ and $a_{t+1} < a_{max}$.

Combined with the fact that $\Pi_L = \Pi$, we can write $\Pi = \delta p \Pi_H + \delta(1-p)\Pi$, or

$$\Pi_H - \Pi = \frac{1-\delta}{\delta p}\Pi.$$

Now, $\Pi > 0$ in the first period of any equilibrium in which the project is funded. Therefore, $\Pi_H - \Pi$ is bounded away from 0 . We conclude that $\Pi_H \geq \bar{\Pi}$ after a finite number of high outputs and so happens with probability 1 as $t \to \infty$. ■

A.5 Proof of Corollary 1

Fastest Repayment Path: We first claim that the specified sequence of w_t and r_t repays the creditor as quickly as possible. Indeed, for any sequence such that $w > 0$ in a period for which $K(U_H, \Pi_H) > 0$, we can decrease w and increase U_H so that $(1-\delta)w + \delta U_H$ remains constant, and increase r so that $w + r$ remains constant. Lemma 1 implies that both worker and creditor earn the same expected surplus following this perturbation. However, perturbing the equilibrium in this way decreases $K(U_H, \Pi_H)$ and therefore repays the creditor faster. Since both (DE) and the upper bound of (LL) bind whenever $\Pi_H < \bar{\Pi}$ in any profit-maximizing equilibrium, the specified sequence is consistent with a profit-maximizing equilibrium.

Slowest Repayment Path: The specified sequence results in $U = 0$ in each period, which clearly maximizes $K(U, \Pi) = \tilde{U}(\Pi) - U$ in that period and so repays the creditor as slowly as possible. The proof of Proposition 3 shows that both (DE) and the upper bound of (LL) bind whenever $\Pi_H < \bar{\Pi}$. This sequence also satisfies the lower bound of (LL), since $w = \frac{c(a)}{p} \geq 0$ and $r = a - w \geq$ because $pa - c(a) \geq 0$ for any $a \leq a_{max}$. Therefore, the specified w_t and r_t are consistent with a profit-maximizing equilibrium. ■

A.6 Proof of Proposition 4

Analogous to the set E, define

$$E^T \equiv \{(U, \Pi) | \exists K \geq 0 \text{ such that } (U, \Pi, K) \text{ are truth-telling equiliwrium payoffs}\}.$$

Define the problem (P) as maximizing (5) subject to (PK-A)-(LL), $(U_H, \Pi_H) \in E^T$ and $(U_L, \Pi_L) \in E^T$, and (TT), with the restriction to $a \in \{0, y\}$. Let $K^T(U, \Pi)$ be the value function for (P).

This proof characterizes $K^T(\cdot)$ with a series of lemmas, then uses that characterization to prove Proposition 4.

Lemma 2 *Define*

$$E_1 \equiv \left\{(U, \Pi) \in E^T | \exists \text{ a solution to (P) with } a = y\right\} \subseteq E^T,$$

and let $E_0 \equiv E^T \backslash E_1$. *Then* $(0, 0) \in E_0$ *and* $K^T(0, 0) = 0$, *so that* $(0, 0)$ *can be supported by liquidating the firm. Moreover, any* $(U, \Pi) \in E_0$ *can be implemented by randomizing between* $(0, 0)$ *and some* $(U', \Pi') \in E_1$.

Proof of Lemma 2

First, note that $(0, 0) \in E_0$, since otherwise (PK-P) would be violated. Consider any (U, Π) that can be implemented with $a = 0$. Then $U_L = \frac{U}{\delta}$, $\Pi_L = \frac{\Pi}{\delta}$, and consequently $K^T(U, \Pi) = \delta K^T\left(\frac{U}{\delta}, \frac{\Pi}{\delta}\right)$. In particular, $K^T(0, 0) = \delta K^T(0, 0)$ and so $K^T(0, 0) = 0$, which can be attained through liquidation.

Consider $(U, \Pi) \in E_0$ with $(U, \Pi) \neq (0, 0)$. If (U, Π) can be implemented with $a = 0$, then $K^T(U, \Pi) = \delta K^T(\frac{U}{\delta}, \frac{\Pi}{\delta}) = \delta K^T(\frac{U}{\delta}, \frac{\Pi}{\delta}) + (1 - \delta) K^T(0, 0) \leq K^T(U, \Pi)$, where the first equality follows by the argument above, the second follows because $K^T(0, 0) = 0$, and the inequality holds because K^T is concave. So $K^T(U, \Pi)$ is linear between $(0, 0)$ and $(\frac{U}{\delta}, \frac{\Pi}{\delta})$ for any $(U,\Pi) \in E_0$. Let (U',Π') be the right endpoint of this linear segment, and note that $(U',\Pi') \in E_1$. Therefore, any $(U,\Pi) \in E_0$ can be implemented by randomizing between liquidation and some $(U',\Pi') \in E_1$.

Finally, if $(U,\Pi) \in E_0$ can be implemented with a such that $0 < \Pr\{a = y\} < 1$, then (U,Π) can also be implemented by randomizing between a point in E_1 and a point that can be implemented with $a = 0$. So by the above argument, such (U,Π) can also be implemented by randomizing between continuation and liquidation. ■

Now, define

$$\begin{aligned} \Pi_{\max} &= py - c; \\ \Pi_f &= \tfrac{p(1-\delta)y}{1-(1-p)\delta}; \\ \tilde{U}^T(\Pi) &= \max\left\{U | (\Pi, U) \in E^T\right\}. \end{aligned}$$

Note that (8) implies that $\Pi_f \leq \Pi_{\max}$.

Lemma 3 *For any $(U,\Pi) \in E^T$,*

1. *$U + \Pi = \Pi_{max}$ if and only if $\Pi \in [\Pi_f, \Pi_{\max}]$ and $U = \tilde{U}^T(\Pi)$;*
2. *If $U + \Pi < \Pi_{\max}$, then $K^T(U,\Pi) + U + \Pi < py - c$.*

Proof of Lemma 3

Suppose that $U+\Pi = \Pi_{\max}$. Then $\Pi_{\max}$ is the maximum feasible total surplus, so $K^T(U,\Pi) = 0$ and hence $U = \tilde{U}^T(\Pi)$. Define Π_f' as the manager's smallest equilibrium payoff such that $\tilde{U}^T(\Pi_f') + \Pi_f' = \Pi_{\max}$, and denote Π_H and Π_L as the associated continuation profits. Note that $a = y$ for $(\tilde{U}^T(\Pi_f'), \Pi_f')$, and so

$$\begin{aligned} \Pi_f' &= p((1-\delta)(y-w) + \delta\Pi_H) + (1-p)\delta\Pi_L \\ &\geq p(1-\delta)y + (1-p)\delta\Pi_f', \end{aligned}$$

where the equality holds by (PK-A) and the inequality follows because (DE) implies $\delta\Pi_H \geq (1-\delta)w$, and $\Pi_L \geq \Pi_f'$ in order for sum of the manager's and worker's payoffs to equal $\Pi_{\max}$.

Rearranging this expression yields $\Pi_f' \geq \Pi_f$.

Now, suppose $\Pi \geq \Pi_f$, and consider the set of stationary strategies such that $a = 1$, $r = 0$ and $w \in \left[\frac{c}{p}, \frac{\delta \Pi_f}{1-\delta}\right]$; it is straightforward to show that these actions can be sustained in a relational contract. With $w = \frac{c}{p}$, the manager earns $\Pi_{\max}$; with $w = \frac{\delta \Pi_f}{1-\delta}$, the manager's payoff is Π_f. Therefore, $\tilde{U}^T(\Pi) + \Pi = py - c$ for any $\Pi \geq \Pi_f$. Combined with the result that $\Pi_f' \geq \Pi_f$, we conclude that $\Pi_f' = \Pi_f$ and that for any $\Pi \in [\Pi_f, \Pi_{\max}]$, $\tilde{U}^T(\Pi) + \Pi = py - c$, which proves part 1.

Next, define

$$z \equiv \min \left\{ U + \Pi | K^T(U, \Pi) + U + \Pi = \Pi_{\max} \right\}.$$

Suppose $z < \Pi_{\max}$, and choose $(U, \Pi) \in E^T$ such that $U + \Pi = z$ and $K^T(U, \Pi) + U + \Pi = py - c$. Then it must be that $a = 1$ with probability 1, and moreover $U_L + \Pi_L + K^T(U_L, \Pi_L) = py - c$. Then summing (PK-A) and (PK-P) implies that

$$\begin{aligned} z &= (1-\delta)(py - c - pr) + \delta p(U_H + \Pi_H - U_L - \Pi_L) + \delta(U_L + \Pi_L) \\ &\geq (1-\delta)(py - c) + \delta(U_L + \Pi_L), \end{aligned}$$

where the inequality follows from (TT). Since $z < py - c$, $U_L + \Pi_L < z$. But $U_L + \Pi_L + K^T(U_L + \Pi_L) = py - c$, yielding a contradiction. ■

Lemma 4 *The following hold:*

1. *For* $\Pi \leq \Pi_f$,
$$\tilde{U}^T(\Pi) = \frac{\tilde{U}^T(\Pi_f)}{\Pi_f}\Pi.$$

2. *For any* $\Pi \in [0, \Pi_{\max}]$, $K^T\left(\tilde{U}^T(\Pi), \Pi\right) = 0$.

3. *For any* (U, Π) *with* $U + \Pi < py - c$, $K^T(U, \Pi) + \Pi$ *is strictly increasing in* Π. *For* $\Pi \geq \Pi_f$, $K^T(U, \Pi) + U$ *is strictly increasing in* U.

Proof of Lemma 4

Part 1: Define $\xi = \max\left\{\frac{U}{\Pi} | (U, \Pi) \in E^T\right\}$ and let (U, Π) be such that $\frac{U}{\Pi} = \xi$. Lemma 2 implies that we can take $(U, \Pi) \in E_1$, and it is immediate from the definition of $\tilde{U}^T(\cdot)$ that we can take $U = \tilde{U}^T(\Pi)$. We claim that $\Pi = \Pi_f$. For any $\Pi > \Pi_f$, $\tilde{U}^T(\Pi) + \Pi = \Pi_{\max}$, and so $\tilde{U}^T(\Pi)$ is strictly decreasing in Π on this range. So $\frac{\tilde{U}^T(\Pi_f)}{\Pi_f} > \frac{\tilde{U}^T(\Pi)}{\Pi}$ for any $\Pi > \Pi_f$.

Consider $\Pi < \Pi_f$. To show this, we note two properties. First, $r = 0$ in any equilibrium with payoffs $(\tilde{U}^T(\Pi), \Pi)$, since otherwise we could decrease r and increase w so that $r + w$ is constant and the worker earns a strictly higher payoff than $\tilde{U}^T(\Pi)$.

Second, we claim that there exists an equilibrium giving $(\tilde{U}^T(\Pi), \Pi)$ in which $\Pi_H > \Pi_f$. Recall that $(\tilde{U}^T(\Pi), \Pi) \in E_1$, and note that either (DE) or the upper bound of (LL) must bind, since otherwise we could increase w and hence increase the worker's payoff. Suppose the upper bound of (LL) is slack, so $w < y$. From the public randomization device, $\tilde{U}^T(\Pi)$ is concave, so $\Pi + \tilde{U}^T(\Pi)$ is increasing in Π (because it is constant for $\Pi \geq \Pi_f$). Therefore, consider increasing w and $\frac{\delta}{1-\delta}\Pi_H$ by the same amount. Doing so holds the manager's payoff constant and gives the worker a higher payoff. So there exists an equilibrium giving $(\tilde{U}^T(\Pi), \Pi)$ in which $w = y$. But then

$$\delta\Pi_H \geq (1-\delta)y > \frac{\delta p(1-\delta)y}{1-(1-p)\delta} = \delta\Pi_f,$$

as desired.

Given these two properties,

$$\begin{aligned}
1+\xi &= \frac{U+\Pi}{\Pi} \\
&= \frac{(1-\delta)(py-c)+\delta p(py-c)+\delta(1-p)(U_L+\Pi_L)}{\Pi} \\
&\leq \frac{(1-\delta+\delta p)(py-c)+(1-p)\Pi_L(1+\xi)}{\Pi} \\
&\leq \frac{(1-\delta+\delta p)(py-c)+(1-p)\Pi_L(1+\xi)}{(1-\delta)py+\delta(1-p)\Pi_L} \\
&\leq \frac{(1-\delta+\delta p)(py-c)}{(1-\delta)py} \\
&= \frac{\tilde{U}^T(\Pi_f)+\Pi_f}{\Pi_f}
\end{aligned} \tag{11}$$

Here, the first equality follows from $\Pi_H > \Pi_f$ and so $\tilde{U}^T(\Pi_H) + \Pi_H = py - c$, the first inequality holds because $\frac{U_L+\Pi_L}{\Pi_L} \leq 1 + \xi$ by definition of ξ, the second inequality follows because (DE) implies that $\Pi \geq (1-\delta)py + \delta(1-p)\Pi_L$, the third inequality holds because $\delta(1-p)\Pi_L \geq 0$, and the final equality holds by definition of Π_f and because $\tilde{U}^T(\Pi_f) + \Pi_f = py - c$.

We conclude that $\frac{\tilde{U}^T(\Pi_f)}{\Pi_f} = \xi$, as desired, which implies part 1 of Lemma 4 because $\tilde{U}^T(\Pi)$ is concave and so $\frac{\tilde{U}^T(\Pi)}{\Pi}$ is decreasing in Π, and strictly so unless $\tilde{U}^T(\cdot)$ is linear.

Part 2: Note that for $\Pi \geq \Pi_f$, Lemma 3 implies that $K^T(\tilde{U}^T(\Pi), \Pi) = 0$. For $\Pi < \Pi_f$, (11) holds with equality only if $\frac{U_L}{\Pi_L} = \xi$. But then $\Pi_L \geq \Pi_f$, implying that $K^T(U_L, \Pi_L) = 0$, and similarly $\Pi_H \geq \Pi_f$ so $K^T(U_H, \Pi_H) = 0$. Since $r = 0$ as well, $K^T(\tilde{U}^T(\Pi), \Pi) = 0$ in this range too.

Part 3: Lemma 3 and the concavity of $K^T(\cdot)$ imply that $K^T(U, \Pi) + U$ is strictly increasing in U for $\Pi \geq \Pi_f$. Similarly, Lemma 3, concavity of K^T, and the fact that $\tilde{U}^T(\Pi)$ is maximized at Π_f imply that $K^T(U, \Pi) + \Pi$ is strictly increasing whenever $U + \Pi < py - c$. ■

Given this characterization, we are prepared to prove our main result.

Proof of Proposition 4

Part 1: Suppose that $\Pi \geq \frac{p(1-\delta)y}{1-(1-p)\delta}$ and $K^T(U,\Pi) > 0$. Since $U+\Pi+K^T(U,\Pi) \leq \Pi_{max} \equiv py-c$, $U+\Pi < \Pi_{max} \equiv py-c$. Part 2 of Lemma 3 therefore implies that

$$K^T(U,\Pi) + U + \Pi < py - c.$$

Now, part 1 of Lemma 3 implies that $K^T(\tilde{U}^T(\Pi),\Pi)+\tilde{U}^T(\Pi)+\Pi = py-c$, where $\tilde{U}^T(\Pi) > U$ by definition. Since

$$K^T(\tilde{U}^T(\Pi),\Pi) + \tilde{U}^T(\Pi) + \Pi > K^T(U,\Pi) + U + \Pi$$

and $K^T(\cdot)$ is weakly concave, we conclude that

$$K^T(U',\Pi) + U' + \Pi > K^T(U,\Pi) + U + \Pi$$

for any $U' > U$.

Part 2: Suppose $U_H + \Pi_H < py - c$. First, we argue that $r + w = y$ whenever $U_H + \Pi_H < py - c$. Note that $\Pi_H < \Pi_f$. Suppose $r + w < y$, and consider an alternative that increases r by $\epsilon > 0$ and increases Π_H by $\frac{1-\delta}{\delta}\epsilon$. For $\epsilon > 0$ sufficiently small, this perturbation is feasible—in particular, Lemma 4 implies $(U_H, \Pi_H + \frac{1-\delta}{\delta}\epsilon) \in E^T$ because $\Pi_H < \Pi_f$—and it continues to satisfy the constraints of (P). Moreover,

$$\delta K^T\left(U_H, \Pi_H + \frac{1-\delta}{\delta}\epsilon\right) + (1-\delta)\epsilon > \delta K^T(U_H,\Pi_H)$$

by part 3 of Lemma 4. So the original equilibrium cannot be on the frontier defined by $K^T(\cdot)$; contradiction.

Next, we show that $w = 0$. Suppose $w > 0$, and consider increasing r and decreasing w by $\epsilon > 0$, and increasing U_H by $\frac{1-\delta}{\delta}\epsilon$. As before, this perturbation satisfies the constraints of

(P). It is also feasible for sufficiently small $\epsilon > 0$, since $\Pi_H > \Pi_f$ from the proof of Lemma 4 and so $U_H < py - c - \Pi_H \leq \tilde{U}^T(\Pi_H)$.

Now, since $\Pi_H > \Pi_f$,

$$\delta\epsilon + \delta K\left(U_H + \frac{1-\delta}{\delta}\epsilon, \Pi_H\right) > \delta K(U_H, \Pi_H)$$

by part 3 of Lemma 4. So the creditor earns a strictly higher payoff in the perturbed equilibrium. Hence, $w = 0$ and $r = y$.

Part 3: By Lemma 2, it suffices to show that when $K(U, \Pi) > 0$, continuation play at some successor history lies in E_0 with positive probability. Suppose not; then $K(U, \Pi) + U + \Pi = py - c$. But then Lemma 3 (part 1) and Lemma 4 (part 2) imply that $K(U, \Pi) = 0$; contradiction. ■

B Data Appendix

B.1 German Administrative Data

We use two datasets from the Federal Data Center (FDZ) of the Institute for Employment Research (IAB) of German Employment Agency: the Linked Personnel Panel (LPP) survey of firms and workers and the IAB Establishment Panel (IAB-EP), a survey of firms. To access the desired waves of both datasets, we combine two FDZ products: LPP-ADIAB Version 1975–2014 and LIAB. LPP-ADIAB is the LPP survey data that is linked to the administrative data of the IAB. The LIAB is the Longitudinal Model (version 1993–2014) of the Linked Employer-Employee Data from the IAB. The data were accessed on-site at the Research Data Centre of the Federal Employment Agency at the Institute for Employment Research (FDZ) and via remote data access at the FDZ. The documentation for LPP-ADIAB is available in Broszeit et al. (2017), and the documentation for LIAB is available in Heining et al. (2016). The webpage of the Federal Data Center (https://fdz.iab.de/en/) contains the questionnaires and further details about both datasets.

The LPP data consists of two waves. In both of them, around 6,000 workers were interviewed, 3,000 of whom were interviewed in both waves. These workers are selected from approximately 1,000 establishments from the IAB-EP sample. We obtain information about leverage from the 2008 wave of the IAB-EP, which does not contain all of the relevant firms. After linking surveyed individuals to our data on leverage, we have around 7,600 observations. We do not otherwise restrict the sample, although the number of observations varies slightly across regressions because some of the variables are missing for some observations.

Using the LPP data, we define 3 outcome variables that we use in Table 2. First, "considers quitting" is based on question "How many times in the past 12 months have you thought about changing your job?". The raw answers are (1) daily, (2) a few times a week, (3) a few times a month, (4) a few times a year, (5) never. We recode the variable by subtracting the value from 6 so that higher numbers correspond to a higher intensity of considering quitting. Second, “commitment” is based on 6 statements from the “commitment” section of the survey. For each question, the respondent employee chooses one of 5 answers: (1) fully applies; (2) largely applies; (3) neutral; (4) does rather not apply; and (5) does not apply at all. An example statement is, “I would be very happy to spend the rest of my career with this organization.” For each statement, we again recode the answers so that higher values correspond to higher commitment. Finally, “job satisfaction” is based on the question “How satisfied are you today with your job? Please answer on a scale from 0 to 10, where 0 means ’totally unhappy’ and 10 means ’totally happy.’” The main independent variable in the worker-level regressions, as well as some of the control variables, come from IAB-EP, which we will discuss further in the next paragraph. Additional controls include the workers’ sex and year of birth, a categorical measure of their educational level, their income level (five quintiles), and an indicator for their contract with the firm having a fixed term (instead of a permanent contract).

The IAB-EP is an annual survey that randomly samples around 16,000 establishments. Firms are surveyed repeatedly, but each year, some firms stop participating and other firms are added. The survey includes modules on firm personnel, investment, business practices, and other topics. Some questions are repeated every year, while others are only asked in selected years. For our regressions, questions about staffing problems are asked every 2 years, while the question about firm leverage was asked only in 2005 and 2008.

Our measure of leverage is based on the question, “Which of the following sources of finance were used for the (reported) total investment? Please indicate how this amount is distributed across the different sources: (...) Private credit (from banks, credit unions, saving banks, enterprises) (...) Approx. .... %”. That is, this measure gives the share of investment in a given year financed with debt, as opposed to current receipts, other equity, and subsidies. Measures of staffing problems are based on the question: “What kind of problems with human resources management do you expect for your establishment/office during the next two years? Please tick where applicable in the list!”, with possible answers including “lack of motivation in the workplace" and "high turnover." Our additional controls from this survey include average wage in the establishment (total wage bill divided by number of employees), relative employment growth (change in employment divided by total employment), type of management (by owner, by professional manager, mixed) and profitability in the recent year, which is measured according to the five-category profitability scale "very good", "good", "satisfactory", "sufficient", or "unsatisfactory." We perform robustness checks with an alternative three-category scale "positive," "approximately balanced," or "negative."

B.2 Amadeus Data

Our regressions on wages and productivity use the Amadeus dataset, which we access using WRDS. Since the required variables are often unavailable for small firms, we include only large and very large companies. A firm is considered large or very large if it meets at least one of the following conditions: more than 150 employees, operating revenue higher than 10 mln EUR, or total assets of 20 mln EUR or more. We use the entire available time period, 1985–2017, but since Amadeus provides no more than 10 years of data for each firm, for most of firms our data comes from between the 2006/2007 and the 2016/17 waves. We keep the consolidated data whenever both the consolidated and unconsolidated data are available. If a firm has more than one observation per year, we use the latest.

Capital is defined as the log of fixed assets, and labor is defined as the log of employee costs. We use operating revenue as a proxy for output since sales entails more missing data. Our proxy for intermediate inputs is the log of material costs, and the industry is given by 2-digits codes from the NACE2 classification. We keep only manufacturing firms, which have NACE2 codes between 10 and 32, and we drop firms that have fewer than 5 yearly observations of capital. To reliably estimate TFP by (industry X country), we keep only those industry-country pairs that have at least 1000 firm-year observations with non-missing capital, labor, output, and intermediate input measures. Our results with OLS-based TFP-R are similar if we instead use a threshold of 100 observations; our other measures, however, are more data-intensive, so they frequently fail to converge if the number of observations is too small.

This restriction greatly reduces our working sample for two reasons. First, not all information is available for all countries. For example, firms in the United Kingdom do not report intermediate outputs, so they are dropped. Second, smaller countries or industries might not have 1000 observations. Nevertheless, all but one industry (tobacco manufacturing) is included for at least one country, and all large European countries, except for the United Kingdom, are included in the final sample. Our final data has 127,703 observations. To ensure that our results are not driven by outliers, we trim both regressands and regressors at the 5th and 95th percentile. Table 4 presents additional summary statistics for our final sample.

Table 4: Additional Summary Statistics

Panel A: Summary Statistics for German Administrative Data:

Firm-Level Data	Mean	Median	St Dev	Min	Max
Leverage	15.4	0	30.4	0	100
Leverage>0	0.250	0	0.433	0	1
Problems with Motivation	0.072	0	0.259	0	1
Problems with Turnover	0.029	0	0.168	0	1
Num of Employees	164.4	21	856.2	0	49723
Employment Growth	-0.094	0	13.916	-3449	1
Average Wage	1530	1454.5	1166.2	0	15000
Profitability	4.03	5	1.63	1	5
Obs Cross-Section (Lev):	14 625				
Obs Panel (Lev):	19 227				
Worker-Level Data					
Considers Quitting	1.59	1	0.92	1	5
Commitment to Firm	3.72	3.83	0.89	1	5
Job Satisfaction	7.54	8	1.71	1	5
Female	0.28	0	0.45	0	1
Year of Birth	1966.6	1965	10.3	1945	1996
Education Level	3.29	3	1.13	1	8
Income Level	2.97	3	1.41	1	5
Fixed-Term Contract	0.050	0	0.218	0	1
Observations	11 809				

Panel B: Summary Statistics for Amadeus Data

	Mean	Median	Std Dev	Min	Max
Operating revenue (mln Euro)	60.0	25.3	125.7	0.5	8277.8
Fixed assets (Mln Euro)	16.1	6.4	27.5	0.3	223.5
Costs of employees (Mln Euro)	9.7	4.1	26.6	0.0	4115.3
Material costs (Mln Euro)	33.3	12.7	76.5	0.0	2733.5
Firm Age (Years)	24.7	23	19.9	1	304
Value Added/Total Assets	0.324	0.289	0.225	-4.402	4.820
TFP-R (GNR)	23.348	23.343	0.676	15.824	26.043
Observations	127703				

Panel A presents summary statistics from LPP and IAB-EP data. Statistics for firm-level variables refer to the full dataset which contains values from 2004-2008 period. For panel regressions, only 2 observations per firm are used (2005/2006 and 2008). For cross-sectional regressions, one observation per firm - averaging values from the whole period - is used. For worker-level data, summary statistics come from the pooled cross-section of workers from two waves of LPP survey. For firm-level data, number of observations refers to number of non-missing leverage observations; for worker data, it refers to all observations among those that are merged with firm-level leverage measure and are used in regressions. Profitability is a categorical variable with 5 possible values, raning from 1 ("poor") to 5 ("very good"). Panel B presents summary statistics from Amadeus data. The sample contains all firms with more than 150 employees, operating revenue higher than 10 million EUR or total assets of 20 million EUR or more. Observations come from 1985-2017, but each firm has at most 10 observations, usually from 2006/07-2016/17.

C Internet Appendix: Collusion and Truth-telling

This section defines an equilibrium refinement that captures collusion between the manager and worker, then shows that Definition 1 is sufficient to incorporate this notion of collusion. This sufficiency argument relies on binary effort.

Definition 2 *A SPE σ^* is a* collusion equilibrium *if, after the creditor accepts the equilibrium formal contract $(R, l(\cdot))$, continuation play maximizes the manager's payoff among all continuation SPE.*

The intuition for a collusion equilibrium follows from the following heuristic timing. Suppose that the manager first negotiates with the creditor to secure a loan. After the creditor signs this formal contract, however, the manager can sit down with the worker and propose a continuation equilibrium. During her negotiations with the creditor, the manager cannot credibly promise to choose an equilibrium in which the worker punishes her for failing to repay the creditor. Therefore, Definition 2 captures the idea that the creditor does not have a seat at the table when the manager and worker decide on a relational contract. Essentially, such equilibria resemble "multi-tier" contracting problems (Tirole (1986); DeMarzo et al. (2005)), with the important differences that the game is infinite-horizon and contracts must be self-enforcing.

One immediate implication of this definition is that liquidation must occur with positive probability whenever the project is funded in a collusion equilibrium. If it did not, then the manager and worker could agree to never repay the creditor for her initial loan. We prove a stronger result: there exists a profit-maximizing truth-telling equilibrium that is also a collusion equilibrium.

Proposition 5 *Consider the model with binary effort from Section 5. There exists a profit-maximizing truth-telling equilibrium that is a collusion equilibrium.*

Proof: Proposition 4 says that there exists a truth-telling equilibrium in which $a_t = y$ in every period until the project is liquidated. Consider this equilibrium and note that it maximizes *total* surplus given the formal contract $l(\cdot)$. It suffices to show that, immediately after the creditor agrees to $l(\cdot)$, no alternative equilibrium gives the manager a strictly higher expected continuation payoff.

Consider the following "ancillary game," which has two players: a firm and a creditor. In each period, the firm chooses a_t, r_t, and w_t, bears the cost ca_t, and earns the sum of the manager and worker's utility. The creditor's actions and payoffs are unchanged. Fix $l(\cdot)$ as in the original game. Since the firm earns $\Pi + U$ in each period, (7) implies that it has no one-shot deviation in r_t. It has no one-shot deviation in a_t either since, regardless of $l(\cdot)$, the actions $a_t = 1$ and $r_t = 0$ generate a strictly higher sum of manager and worker utilities than $a_t = r_t = 0$. Finally, it has no deviation in w_t, which does does not affect the sum of the manager's and worker's payoff. The one-shot deviation principle applies to this ancillary game, so the firm's payoff is maximized by choosing a_t, r_t, and w_t as specified in the collusive-proof equilibrium.

Now, return to the three-player game. The preceding argument implies that after the creditor agrees to $l(\cdot)$, $\Pi + U$ is maximized by following the equilibrium strategy. However, $U = 0$ immediately after the creditor agrees to $l(\cdot)$. Since $U \geq 0$, Π is bounded above by $\Pi + U$, and moreover, we have argued that $\Pi = \Pi + U$ at the point where $\Pi + U$ is maximized. Therefore, Π is maximized by following the equilibrium, so there exists no alternative equilibrium that generates strictly a higher profit given $l(\cdot)$. The manager is willing to choose $l(\cdot)$ in equilibrium if all other contracts lead to the worker choosing $a_t = 0$ in every t. We conclude that this profit-maximizing truth-telling equilibrium is a collusion equilibrium, as desired. ■

Proposition 5 exhibits a profit-maximizing truth-telling equilibrium that also satisfies Definition 2. However, this result does not say that this equilibrium is a *profit-maximizing* collusion equilibrium. It is in this sense that Definition 1 is a sufficient but *not* a necessary condition for a collusion equilibrium—the manager can certainly do at least as well in a collusion equilibrium, but it is an open question whether she could do strictly better.

D Additional Empirical Results

This appendix presents a number of additional specifications and robustness checks.

Exploring Heterogeneity in Morale Effects

We begin by exploring how our effects vary with (i) whether the firm is in an industry with high level of investment, and (ii) the firm's profitability. Table 5 interacts leverage with binary measures of investment intensity and high profit. We find little evidence of heterogeneity along either of these directions. Indeed, the point estimates on profitability suggest that if anything, debt is correlated with a *larger* increase in motivation problems at highly profitable firms, though these estimates are not significant.

Productivity Regressions with German Employment Agency Data

Next, we re-run the productivity and wages regressions (Table 3) using the data from the German Employment Agency. This data includes firm-level information about average wages. Given the limitations of this data, we use two measures of productivity: TFP-R and TFP-VA. TFP-R is computed based on revenue reported by the firm. TFP-VA is based on value added, constructed by multiplying revenue by one minus the share of intermediate costs. Labor input is measured with total wages, while capital input is constructed using the perpetual inventory method based on firm's past investment flows and industry-specific depreciation rates of capital. Both TFP measures are calculated using OLS regressions in three broad economic sectors: (1) agriculture, mining, and manufacturing; (2) low-skill services (e.g. retail); (3) high-skill services (e.g. finance). The relatively small dataset prevents us from relying on more sophisticated TFP calculations.

Table 6 presents both cross-sectional and panel regressions using these measures of wages and productivity. The results are consistent with Table 3: in both the cross section and the

	Heterogeneity by Industry Capital Intensity				Heterogeneity by Profitability				
	Problem with Motivation		Problem with Turnover		Problem with Motivation			Problem with Turnover	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Leverage	0.032***	0.0204+	0.009+	0.0309***	0.031***	0.0182		0.009+	0.0267**
	(0.009)	(0.015)	(0.005)	(0.013)	(0.009)	(0.015)		(0.006)	(0.012)
Leverage	0.015	-0.017	0.014	-0.0308					
x High Investment	(0.024)	(0.030)	(0.013)	(0.024)					
Leverage					0.018	0.005		0.015	-0.0090
X High Profits					(0.020)	(0.020)		(0.013)	(0.0160)
High Leverage							0.205**		
							(0.010)		
High Leverage							0.006		
X High Profits							(0.013)		
N	9 677	17 522	9 677	17 522	9 677	17 522	17 522	9 677	17 522
Industry FE	✓		✓		✓			✓	
Firm FE		✓		✓		✓	✓		✓

Table 5: Heterogeneity by Industry Capital Intensity and Profitability

The table presents the regressions of measures of morale on leverage and its interactions with indicator for high investment and high profitability. Dependent variables and leverage measures are defined analogously to the main specification, and the same set of control variables is included (size, profitability, age, average wage). High investment is defined as industry-level ratio of investment per worker being above median. High profitability is defined as a binary indicator for a firm reporting significantly positive profits. In the panel regression columns, standard errors are clustered on the firm level. (***) denotes significance at 1% level, (**) at 5%, (*) at 10% and (+) at 15% level.

	TFP-R		TFP-VA		Avg Wage	
	(1)	(2)	(3)	(4)	(5)	(6)
Leverage	-0.083***	-0.031*	-0.164***	-0.075**	-125.2***	32.2
	(0.015)	(0.018)	(0.031)	(0.035)	(21.4)	(33.0)
N	7 898	11 975	7 895	11 956	9 697	19 224
Industry FE	✓		✓		✓	
Firm FE		✓		✓		✓

Table 6: The relationship between debt, wages, and productivity in the German Employment Agency data
The table presents the regressions of measures of productivity on firm leverage. Leverage is defined analogously to the main specification, and the same set of control variables is included (size, profitability, age, average wage). Dependent variables are Total Factor Productivity measures based on firm revenues and value added. Both measures are computed in an OLS regression within 3 broadly defined sectors. In the panel regression columns, standard errors are clustered on the firm level. (***) denotes significance at 1% level, (**) at 5%, (*) at 10% and (+) at 15% level.

panel, leverage is correlated with lower productivity. Leverage is also negatively correlated with wages in the cross-section, although there is no significant correlation in the panel specification, possibly because of significant wage rigidity in Germany. Because our panel includes only 2 waves, it is not possible to include lagged leverage in this specification.

Non-Linear Effects of Debt

Our next regressions test whether leverage has a non-linear effect on morale, wages, and productivity. To explore non-linearities in the wage and leverage effects, Table 7 incorporates quadratic terms for (contemporaneous and lagged) leverage, as well as regressions that break the leverage effect into quartiles. Overall, our results are similar; some of the quadratic terms are significant, but there is no obvious interpretation of these non-linearities, and the quartile regressions exhibit largely monotone effects.

Morale Results: Alternative Definitions of Leverage

This section examines the correlation between worker morale and two alternative measures of leverage. The first measure, "Debt/Worker," divides a firm's debt by the number of

	Amadeus Data				German Administrative Data			
	ΔL(Wage)		ΔTFP		Problem with Motivation		Problem with Turnover	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
ΔLeverage	-0.089***		-0.421***		0.030	0.050	0.0429**	0.028
	(0.017)		(0.0654)		(0.028)	(0.049)	(0.0182)	(0.038)
$(\Delta\text{Leverage})^2$	0.402*		1.117***		0.005	-0.036	-0.037*	-0.005
	(0.1936)		(0.3080)		(0.031)	(0.057)	(0.020)	(0.046)
ΔLeverage (t-1)	-0.015		-0.066***					
	(0.0103)		(0.0109)					
$(\Delta\text{Leverage (t-1)})^2$	-0.331*		-0.017					
	(0.1917)		(0.165)					
ΔLeverage Q2		-0.00441***		-0.0185***				
		(0.00154)		(0.00251)				
ΔLeverage Q3		-0.00690**		-0.0378***				
		(0.00293)		(0.00492)				
ΔLeverage Q4		-0.00977***		-0.0476***				
		(0.00206)		(0.00726)				
N	127 703	127 703	127 703	127 703	9 677	17 522	9 677	17 522
Clustering	Industry		Industry					
Industry X Year FE	✓	✓	✓	✓				
Industry FE					✓		✓	
Firm FE						✓		✓

Table 7: Non-linear leverage effects on morale, wage, and productivity

The table presents the regressions of various outcomes on non-linear terms measuring leverage. Dependent variables and leverage are defined analogously to the main specification, except that non-linear terms are also included. Columns 1, 3, and 5-8 included a square of both contemporaneous and lagged leverage measure. Columns 2 and 4 substitute continuous leverage measure with binary indicators for 4 leverage quartiles (the bottom quartile is an omitted category). All columns include the same control variables as the corresponding main specifications. In the panel regression columns, standard errors are clustered on the firm level. (***) denotes significance at 1% level, (**) at 5%, (*) at 10% and (+) at 15% level.

employees. The second measure, "Debt/Sales," divides a firm's debt by its total sales.

Table 8 presents results for our firm-level measures of morale. Consistent with our mechanism, these regressions show that higher debt levels (by either measure) are correlated with lower morale. The sign of the coefficients in our worker-level results, presented in Table 9, are also broadly consistent with our mechanism, though the coefficients are typically not significant.

Worker-Level Results Using the Full Scale of Outcomes

This section re-runs our worker-level morale regressions using the full reported scale of each morale measure, rather than a binary indicator of the outcome being above a threshold. These results are presented in Table 10; they are consistent with those presented in Table 2, although once we add controls, the coefficients in Columns 4 and 6 are either insignificant or marginally significant.

Testing for Granger Causality in Wage and Leverage Regressions

Finally, we test whether debt Granger-causes lower productivity and wages. Table 11 shows that we find Granger causality in both directions—from debt to productivity and wages, as well as from productivity and wages to debt—although the result is substantially stronger for the direction consistent with our mechanism: debt Granger-causes changes in productivity and wages.

	Debt per Worker				Debt to Sales			
	Problem with Motivation		Problem with Turnover		Problem with Motivation		Problem with Turnover	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Debt/Worker	0.0246*	0.0511**	0.006	0.0321*				
	(0.014)	(0.023)	(0.009)	(0.018)				
Debt/Sales					0.0352**	0.0550+	0.0148	0.0548**
					(0.017)	(0.034)	(0.013)	(0.028)
N	8 608	15 527	8 608	15 527	6 774	11 029	6 774	11 029
Industry FE	✓		✓		✓		✓	
Firm FE		✓		✓		✓		✓

Table 8: Robustness of Leverage Definition: Debt per Worker and Debt/Sales Ratio. In the cross-sectional columns, standard errors are clustered by industry.

The table presents the regressions of morale outcomes on alternative measures of leverage. Dependent variables are defined analogously to the main specification, and the same control variables are included (size, profitability, age, average wage). In columns 1-4, leverage is measured by the amount of debt per worker, where the monetary value of debt is calculated as our standard measure of leverage in IAB data (share of debt in investment flows) multiplied by the amount of investment reported. In columns 5-8, we use the ratio of debt to total sales. In the panel regression columns, standard errors are clustered on the firm level. (***) denotes significance at 1% level, (**) at 5%, (*) at 10% and (+) at 15% level.

	Seriously Considering Quitting		High Commitment to Firm		High Job Satisfaction	
	(1)	(2)	(3)	(4)	(5)	(6)
Debt per Worker	1.41		-2.37**		0.005	
($/10^8$)	(1.38)		(1.17)		(1.18)	
Debt/Sales		0.283		-0.758		0.355
		(1.122)		(0.822)		(0.739)
N	7 399	6 625	7 408	6 633	7 407	6 632

Table 9: Robustness of Leverage Definition: Debt per Worker and Debt/Sales Ratio
The table presents the regressions of worker morale outcomes on alternative measures of leverage. Dependent variables are defined analogously to the main specification, and the same control variables are included (sex, age, education, income level, contract type, firm average wage, firm age, and firm profitability status). In columns 1-4, leverage is measured by the amount of debt per worker, where the monetary value of debt is calculated as our standard measure of leverage in IAB data (share of debt in investment flows) multiplied by the amount of investment reported. In columns 5-8, we use the ratio of debt to total sales. Standard errors are clustered on the firm level. (***) denotes significance at 1% level, (**) at 5%, (*) at 10% and (+) at 15% level.

	Considers Quitting		Commitment to Firm		Job Satisfaction	
	(1)	(2)	(3)	(4)	(5)	(6)
Leverage	0.227***	0.180***	-0.157**	-0.086	-0.244**	-0.182+
	(0.069)	(0.066)	(0.070)	(0.072)	(0.113)	(0.115)
N	7 637	7 399	7 647	7 408	7 645	7 407
Add. Controls		✓		✓		✓

Table 10: Worker-Level Impacts with the Full Scale of Outcomes
The table presents the regressions of worker morale outcomes on firm leverage leverage. Leverage is defined in the same way as in the main specification, and the same control variables are included. Dependent variables are defined based on the same workers' responses as binary measure in the main specification, but full scale of outcomes is used. In columns 1-4, the dependent variables vary between 1 and 5, while in columns 5-6 between 1 and 10. Standard errors are clustered on the firm level. (***) denotes significance at 1% level, (**) at 5%, (*) at 10% and (+) at 15% level.

Equation	Excluded Var	Chi-Sq	P-value
ΔTFP	ΔLev	53.5	0.000
ΔLev	ΔTFP	4.2	0.040
$\Delta Log(Wage)$	ΔLev	10.2	0.001
ΔLev	$\Delta Log(Wage)$	5.8	0.016

Table 11: Granger-Causality Tests for the link between Debt, Wages, and Productivity
The table presents results of Granger-causality tests. Chi-Sq column shows the Chi-square statistic for a test of the null hypothesis that excluded variable does not Granger-cause Equation variable. For example, row 1 demonstrates that changes in leverage do Granger-cause changes in productivity. Tests are based on panel vector autoregressions with 135 745 observations and 28 299 panels (firms), estimated with GMM method.

E Internet Appendix: Truth-Telling with Continuous Effort

This section considers truth-telling equilibria in the game with continuous effort.

Consider the model from Section 3, which allows effort to be any non-negative number, $a_t \geq 0$. With an abuse of notation, let $K^T(U, \Pi)$ be the truth-telling equilibrium payoff frontier for the creditor, given worker and manager payoffs U and Π, respectively. Then $K^T(\cdot)$ solves problem (P'), defined as maximizing (5) subject to (PK-A)-(LL), (TT), and

$$(U, \Pi) \in E^T,$$

where E^T is defined analogously to Section 5.

Define $\tilde{U}^T(\cdot)$ analogously to the proof of Proposition 4, let $\Pi_{max} \equiv pa_{max} - c(a_{max})$, and define

$$\Pi_f \equiv \frac{p(1-\delta)a_{max}}{1-(1-p)\delta}.$$

We prove the following result.

Proposition 6 *There exists a non-empty, open set $\mathcal{B} \subseteq E^T$ such that $K^T(U, \Pi) = K(U, \Pi)$ if and only if $(U, \Pi) \in \mathcal{B}$, and otherwise $K^T(U, \Pi) < K(U, \Pi)$. Moreover,*

1. ***Frontload creditor payments when truth-telling changes behavior:*** *Whenever (U, Π) is such that $\Pi_H < \Pi_f$, $b + r = y$. If $(U_H, \Pi_H) \notin \mathcal{B}$, then $b = 0$ and $r = y$.*

2. ***Productivity increases w/ manager's payoff:*** *There exists $\bar{\Pi} : \mathbb{R}_+ \to \mathbb{R}_+$ such that $s + K(s - \Pi, \Pi)$ is strictly increasing in Π for $\Pi < \bar{\Pi}(s)$ and is constant for $\Pi \geq \bar{\Pi}(s)$.*

The equilibrium payoff frontier can be split into two regions. If $(U, \Pi) \in \mathcal{B}$, then it is as if (TT) does not bind, in which case equilibrium payoffs are identical to payoffs without this constraint. If $(U, \Pi) \notin \mathcal{B}$, then (TT) constrains equilibrium play and leads to lower equilibrium payoffs. Intuitively, (TT) binds when the creditor is owed money. Consequently, and as in Section 5, part 1 of Proposition 6 says that the manager uses the entire output to pay the creditor whenever continuation payoffs lie in $\mathcal{B}$, so that $r = y$ and worker wages are strictly backloaded. Part 2 of this result says that as the manager repays the creditor, productivity (as measured by total continuation surplus) increases, and strictly so unless Π is large.

Together, these results confirm that the main takeaways from Propositions 3 and 4 hold in this setting: worker pay is backloaded in equilibrium, and debt depresses morale and leads to productivity dynamics.

Proof of Proposition 6

We first state a result analogous to Lemma 2.

Lemma 5 *Define*

$$E_1 \equiv \left\{(U, \Pi) \in E^T | \exists \text{ a solution to (P) with } a > 0\right\} \subseteq E^T,$$

and let $E_0 \equiv E^T \backslash E_1$. *Then* $(0, 0) \in E_0$ *and* $K^T(0, 0) = 0$, *so that* $(0, 0)$ *can be supported by liquidating the firm. Moreover, any* $(U, \Pi) \in E_0$ *can be implemented by randomizing between* $(0, 0)$ *and some* $(U', \Pi') \in E_1$.

We omit the proof of this lemma, which follows very similar lines to the proof of Lemma 2.

Lemma 6 *The following hold:*

1. $K^T(U, \Pi) = 0$ *whenever* $U = \tilde{U}^T(\Pi)$.

2. *For all* $\Pi \in [0, \Pi_{max}]$, $\tilde{U}^T(\Pi) = \tilde{U}(\Pi)$.

Proof of Lemma 6

Part 1 follows the same argument as the proof of Lemma 1, since the perturbation used there decreases r and so relaxes (TT).

For part 2, for any $(U, \Pi) \in E^T$ with $U = \tilde{U}^T(\Pi)$, we have $K(U, \Pi) = 0$ by part 1. Consequently, $\tilde{U}_L(\Pi_L) = U_L$ and $\tilde{U}_H(\Pi_H) = \Pi_H$, since otherwise $pK(U_L, \Pi_L) + (1 - p)K(U_H, \Pi_H) > 0$. Then $r = 0$ in all subsequent periods. But in the relaxed problem with $r_t = 0$ and without (TT), yields $\tilde{U}^T(\Pi) = \tilde{U}(\Pi)$. In this relaxed problem, $U_H + \Pi_H \geq U_L + \Pi_L$ and so the solution to the relaxed problem satisfies (TT) as well. ■

Lemma 7 *The following hold:*

1. $K^T(U, \Pi) \leq K(U, \Pi)$ *for all* $(U, \Pi) \in E^T$.

2. $K^T(U, \Pi) < K(U, \Pi)$ *for all* $(U, \Pi) \in E^T$ *such that* $U < \tilde{U}^T(\Pi)$ *and* $\Pi \geq \Pi_f$.

3. *For each* $\Pi < \Pi_f$, *there exists* $g(\Pi) < \tilde{U}^T(\Pi)$ *such that* $K^T(U, \Pi) = K(U, \Pi)$ *for all* (U, Π) *satisfying* $U \geq g(\Pi)$.

Proof of Lemma 7

Part 1: By Lemma 5, it suffices to show this for $(U, \Pi) \in E_1$. Note that $K^T(\cdot)$ satisfies the Blackwell sufficient condition and so can be obtain through a sequence of approximations. Let $K_0^T(U, \Pi) = 0$ for all $(U, \Pi) \in E_1$, and for all $s > 0$, define

$$K_s^T(U, \Pi) = \Gamma^T K_{s-1}^T(U, \Pi),$$

where Γ^T is the operator induced by the problem (P'). Then $K^T(U, \Pi) = \lim_{s \to \infty} K_s^T(U, \Pi)$.

Similarly, let $K_0(U, \Pi) = 0$ for all $(U, \Pi) \in E$, and define

$$K_s(U, \Pi) = \Gamma K(U, \Pi)$$

where Γ is the operator induced by the SPE problem. Then for all $s \geq 0$, $K_s(U,\Pi) \geq K_s^T(U,\Pi)$ because Γ^T entails strictly more constraints than Γ. Consequently, $K(U,\Pi) \geq K^T(U,\Pi)$.

Part 2: Suppose $U < \tilde{U}^T(\Pi)$ and $\Pi \geq \Pi_f$. In this case, $K(U,\Pi) + U + \Pi = \Pi_{max}$ by the proof of Proposition 3. Therefore, if we define

$$z \equiv \min\left\{U + \Pi | K^T(U,\Pi) + U + \Pi = \Pi_{max}\right\},$$

then it suffices to show that $z = \Pi_{max}$.

Suppose to the contrary that $z < \Pi_{max}$, and choose (U,Π) such that $U + \Pi = z$ and $K^T(U,\Pi) + U + \Pi = \Pi_{max}$. Then it must be that $K^T(U_L,\Pi_L) + U_L + \Pi_L = \Pi_{max}$. However, summing (PK-A) and (PK-P) and applying (TT) yields

$$\begin{aligned} U + \Pi &= (1-\delta)(pa_{max} - c(a_{max}) - pr) + \delta p(U_H + \Pi_H - U_L - \Pi_L) + \delta(U_L + \Pi_L) \\ &\geq (1-\delta)(pa_{max} - c(a_{max})) + \delta(U_L + \Pi_L). \end{aligned}$$

Since $U+\Pi = z < \Pi_{max}$, $U_L + \Pi_L < z$. But $K^T(U_L,\Pi_L) + U_L + \Pi_L = \Pi_{max}$, which contradicts the definition of z. But it is clear that $z \leq \Pi_{max}$, so $z = \Pi_{max}$.

Part 3: Note that $K^T(U,\Pi) \leq K(U,\Pi)$ and $\frac{\partial K}{\partial U} = -1$ by Lemma 1. Since $K^T(\cdot)$ is concave, it therefore suffices to show that, for each $\Pi < \Pi_f$, there exists some $g < \tilde{U}^T(\Pi)$ such that $K(g,\Pi) = K^T(g,\Pi)$.

Proposition 3 implies one solution to the problem without (TT) is

$$\begin{aligned} y &= \tfrac{(1-\delta(1-p))}{(1-\delta)p}\Pi \\ r &= \tfrac{\delta(1-p)(\tilde{U}(\Pi)-g)}{1-\delta(1-p)}, \\ b &= a - r \end{aligned}$$

$(U_L, \Pi_L) = (U, \Pi)$, and $(U_H, \Pi_H) = \left(\tilde{U}(\frac{1-\delta}{\delta}a), \frac{1-\delta}{\delta}a\right)$.

Since $\Pi < \Pi_f$, this solution satisfies $\Pi_H + U_H - \Pi_L - U_L > 0$, independent of g. Therefore, (TT) is satisfied for g sufficiently close to $\tilde{U}(\Pi)$, so this solution also solves (P'). Hence, $K^T(g, \Pi) = K(g, \Pi)$. ■

Now, we can extend the function $g(\cdot)$ by setting $g(\Pi) \equiv 0$ for $\Pi \geq \Pi_f$, so that $K^T(U, \Pi) = K(U, \Pi)$ if and only if $U \geq g(\Pi)$.

Lemma 8 *For any $(U, \Pi) \in E_1$, $U' > U$, and $\Pi' > \Pi$,*

1. *If $(U', \Pi) \in E_1$, then $K^T(U', \Pi) + U' \geq K^T(U, \Pi) + U$, and strictly so if $U' < g(\Pi)$;*

2. *If $(U, \Pi') \in E_1$, then $K^T(U, \Pi') + \Pi' > K^T(U, \Pi) + \Pi$ unless $K^T(U, \Pi) = 0$.*

Proof of Lemma 8

Since $K^T(\cdot)$ is concave, it suffices to establish these properties at (U, Π) satisfying $U = \tilde{U}^T(\Pi)$.

Part 1: This result immediately follows from two facts: (i) $K(U, \Pi) + U$ is constant in U by Lemma 1, and (ii) $K^T(U, \Pi) < K(U, \Pi)$ for all (U, Π) such that $U \leq g(\Pi)$.

Part 2: For this property, it suffices to consider $\Pi \geq \arg\max_{\Pi} \tilde{U}^T(\Pi)$. Note that if $\Pi > \Pi_f$ and $U = \tilde{U}^T(\Pi)$, $K(U, \Pi) + \Pi$ is constant in Π. But $K^T(U, \Pi) < K(U, \Pi)$ whenever $K^T(U, \Pi) > 0$ in this range, so the result obtains.

For $\Pi \leq \Pi_f$, recall that $\tilde{U}^T(\Pi) + \Pi$ is strictly increasing in Π. Therefore, holding U fixed at $\tilde{U}^T(\Pi)$ and applying Lemma 1 implies that $\Pi + K(\Pi, U)$ is strictly increasing in Π. Since $K^T(U, \Pi) \leq K(U, \Pi)$, we conclude that $K^T(U, \Pi) + \Pi$ is also strictly increasing in Π. ■

We are now prepared to prove the two parts of Proposition 6.

Proof of Proposition 6, Part 1

It suffices to consider $(U, \Pi) \in E_1$. Suppose $\Pi_H < \Pi_f$.

First, we consider the case with $U_H < \tilde{U}^T(\Pi_H)$. Suppose to the contrary that $r + b < y$, and consider the perturbation $r' = r + \frac{\delta}{1-\delta}\epsilon$, $\Pi_H' = \Pi_H + \epsilon$, with all other variables remaining the same. This perturbation satisfies the constraints of (P'), and in particular is feasible for sufficiently small $\epsilon > 0$ because $U_H < \tilde{U}^T(\Pi_H)$. But

$$\delta p \epsilon + \delta p \left(K^T(U_H, \Pi_H + \epsilon) - K^T(U_H, \Pi_H)\right) > 0$$

by part 2 of Lemma 8. Contradiction of $K^T(U, \Pi)$ maximizing the creditor's payoff given (U, Π).

Next, suppose $U_H = \tilde{U}^T(\Pi_H)$. Then it must be that $\Pi_H > 0$, since otherwise $U_H = U_L = \Pi_L = 0$, so $a = 0$ and hence $(U, \Pi) \notin E_1$. If $b + r < y$, consider the alternative with $b' = b + \frac{\delta}{1-\delta}\epsilon$, $U_H' = U_H - \epsilon$, and $\Pi_H' = \Pi_H + \epsilon$. This change continues to satisfy the constraints of (P'). Moreover, it is feasible and strictly increases the creditor's payoff because $\tilde{U}^T(\Pi) + \Pi$ is strictly increasing in Π for $\Pi < \Pi_f$. Contradiction of $K^T(U, \Pi)$ maximizing the creditor's payoff.

Now, we have already argued that for any $(U, \Pi) \in \mathcal{B}$, $U < g(\Pi)$ and hence$\Pi < \Pi_f$. Therefore, $r + b = y$ by the previous argument. Now suppose that $b > 0$, and consider the perturbation $r' = r + \frac{\delta}{1-\delta}\epsilon$, $b' = b - \frac{\delta}{1-\delta}\epsilon$, and $U_H' = U_H + \epsilon$, with all other variables remaining the same.

For small enough $\epsilon > 0$, this perturbation is feasible and continues to satisfy the constraints of (P'). Moreover, the creditor's payoff increases by

$$\delta p \epsilon + \delta p \left(K^T(U_H + \epsilon, \Pi_H) - K^T(U_H, \Pi_H)\right) > 0,$$

where the inequality holds by part 1 of Lemma 8, since $U_H < g(\Pi_H)$. ■

Proof of Proposition 6, Part 2

By Lemma 5, $K^T(U,\Pi) = 0$ when $U = \tilde{U}^T(\Pi)$. Hence, for all s, $K(s-\Pi,\Pi)$ is minimized at Π such that $s = \tilde{U}^T(\Pi) + \Pi$.

Define

$$\begin{aligned} k(s) &= \max\{K(U,\Pi)|U+\Pi = s\} \\ u(s) &= \min\{U|K(U,\Pi) = k(s) \text{ and } U+\Pi = s\}. \end{aligned}$$

Concavity of $K(\cdot)$ along the line segment $U + \Pi = s$ implies that it suffices to rule out $u(s) > 0$. Suppose to the contrary that $u(s) > 0$ for some s, and let $u^* = \max_s\{u(s)\}$. Let the associated payoffs be $(u^*, \pi^*) \in E^T$ and the surplus level be s^*.

Given $u^* > 0$, $b > 0$ because otherwise we could decrease the payment to the worker and continue to satisfy the constraints of (P'), which would violate the definition of u^*. Given that $b = 0$, (PK-A) implies that

$$u^* = (1-\delta)(-c(a)) + \delta\left(pU_H + (1-p)U_L\right).$$

Define $s_L = U_L + \Pi_L$. We claim that $U_L \leq u(s_L)$. If instead $U_L > u(s_L)$, then we can perturb (U_L, Π_L) to $(U_L - \epsilon, \Pi_L + \epsilon)$ to decrease the worker's payoff while increasing the manager's creditor's payoff and continuing to satisfy the other constraints of (P'). This perturbation again violates the definition of u^*. By a very similar argument, we can show that $U_H \leq u(s_H)$ for $s_H = U_H + \Pi_H$.

Then

$$\begin{aligned} u^* &\leq (1-\delta)(-c(a)) + \delta\left(pu(s_H) + (1-p)u(s_L)\right) \\ &\leq (1-\delta)(-c(a)) + \delta u^*, \end{aligned}$$

where the first inequality follows by the previous paragraph, and the second inequality follows because $u^* = \max_s\{u(s)\}$. Therefore, $u^* \leq -c(a)$, which is a contradiction of (IC). So $u(s) = 0$ for all s. ■