

# Morale and Debt Dynamics

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## **Abstract**

Financial obligations make it more difficult for firms to motivate their employees. Using administrative data, we document that a firm's financial leverage is negatively related to measures of employee morale, wages, and productivity. To explore these facts, we build a dynamic model of a wealth-constrained manager who simultaneously repays a creditor and motivates a worker. If the manager cannot commit to payments, then indebted managers are less willing to pay promised rewards, leading to low worker effort. In profit-maximizing equilibria, effort and wages increase as outstanding debt decreases, although even repaid debts can have lingering effects on effort.

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# 1 Introduction

Firms borrow money to pursue new opportunities and grow, but even an attractive-looking investment is likely to fail unless employees work hard at implementing it. A successful firm must, therefore, motivate its workers even as it repays its debts. However, firms cannot necessarily commit to output-contingent payments, because, for instance, they might face the *ex post* temptation to divert revenue into privately beneficial but unprofitable projects.<sup>1</sup> Hence, both incentives and repayments must be credible in the context of the firm’s ongoing relationships.<sup>2</sup>

In this paper, we show how debts and other financial obligations constrain a manager’s relationships with her workers, which leads to low compensation, low worker morale, and low productivity. We argue that an indebted manager is less willing to follow through on promised rewards to her workers, who are, in turn, less willing to strive for exemplary performance. The result is “bad morale,” in the form of low effort that persists as long as the firm has outstanding debt and might persist even longer. The manager optimally prioritizes repaying her debts to minimize its negative effects on effort; as she does so, her promises to workers become more credible and performance increases. Thus, financial obligations depress morale and lead to productivity dynamics even when a firm is far from bankruptcy. We document several stylized facts that are consistent with this mechanism; specifically, more highly leveraged firms tend to have lower employee morale, wages, and productivity.

As an example of how borrowing constrains relational incentives, consider Lincoln Electric’s decision to significantly increase its leverage in the early 1990s. At the time, Lincoln Electric was struggling to recover from a devastatingly unprofitable international expansion. The company paid smaller discretionary bonuses to its U.S. workers to prioritize repaying its mounting debts, a decision that damaged employee morale and threatened to undermine its

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<sup>1</sup>Shleifer and Vishny (1997) discusses the theory and evidence for these kinds of commitment problems in credit relationships, while Malcomson (2013) discusses similar commitment problems in agency relationships.

<sup>2</sup>See Bull (1987) and Levin (2003), which discuss commitment problems in agency relationships, and Aghion and Bolton (1992), Hart (1995), and Holmstrom and Tirole (1997), which discuss commitment problems in credit relationships.

strong relational incentive system. In a radical departure from the company’s cooperative worker-manager relations, Lincoln workers openly voiced their displeasure regarding small bonuses, while managers expressed fears that the entire incentive system might “unravel” if they did not make good on promised rewards (Feder (1994); Hastings (1999)). Consistent with this example, firms that treat their employees fairly, and firms that tend to rely on relational contracts, tend to maintain low debt ratios (Bae et al. (2011), Fahn et al. (2017)).

We begin by documenting several stylized facts about the relationship between debt and manager-worker relationships. The fundamental problem with empirically analyzing employee morale is that most datasets measure neither morale nor non-contractible effort. Therefore, we employ administrative data from Germany that contains both firm- and worker-level assessments of employee morale. Consistent with our mechanism, we show that a firm’s leverage is negatively related to various measures of morale, even conditional on self-reported, categorical measures of profitability. We then use a broader firm-level panel to show that increases in a firm’s book leverage are correlated with decreases in wages and productivity and that these relationships are strongest in labor-intensive industries and in countries with poor legal enforcement.

These facts lead us to explore a theory of how a firm’s financial obligations might constrain the credibility of its promises to workers. In our model, a liquidity-constrained manager borrows money from a creditor to fund a project and then repeatedly motivates a worker to exert effort. The manager uses realized profits to repay the creditor and compensate the worker, but she cannot commit to these payments because, for instance, she can divert or misuse cash. Promised payments are, instead, made credible by the threat that the worker might shirk their duties, or the creditor might liquidate the project, if they are not paid.

The manager is willing to pay both the creditor and the worker because they will otherwise deny her future profits, which means that she will renege on *both* parties if she finds her *aggregate* promises too onerous. An indebted manager must promise large payments to the creditor, which limits the extent to which she can credibly reward the worker. Debt,

therefore, depresses both wages and effort.

In profit-maximizing equilibria, we show that effort decreases the sum of the worker's and creditor's continuation payoffs. When the manager first takes on a loan, this *aggregate obligation* is large and effort is low. The manager optimally prioritizes repaying the creditor; as she does so, effort stochastically increases until it converges to a steady-state level that is independent of the initial loan. Worker compensation is weakly backloaded while the manager repays her debts. If compensation is strictly backloaded, then the agent's effort depends on both current and past debt payments, so debt might temporarily depress worker morale even after it has been repaid.

One feature of profit-maximizing equilibria is that they require the worker to punish the manager for renegeing on *any* payment, even a payment to the creditor. Such punishments are optimal but potentially impractical since the manager and the worker then have a strong incentive to collude, jointly renege on the creditor, and split the resulting proceeds. We show that the creditor can deter this kind of collusion by threatening to liquidate the firm if it is not promptly repaid. However, liquidation is inefficient, so the threat of it further increases the implicit cost of borrowing. We show that this additional inefficiency gives the manager another reason to delay paying the worker until she has repaid all of her debt. Thus, if the manager and worker can collude, then debt can have an immediate effect on firm survival in addition to its lingering effects on compensation.

## **Related Literature**

Our model applies a relational contracting model (Bull (1987); MacLeod and Malcolmson (1989); Levin (2002); Levin (2003)) to a manager who simultaneously manages relationships with both workers and creditors. The most closely related papers in this literature are Hennessy and Livdan (2009) and Fahn et al. (2018), both of which propose models in which borrowing constrains worker relationships. However, these papers focus on stationary equilibria, while our focus is on the dynamics of productivity and pay. Recently, relational

contracting papers have considered dynamics that arise from liquidity constraints (Fong and Li (2017)), private information (Halac (2012); Malcomson (2016)), or subjective evaluation (Fuchs (2007)). As in our analysis of collusion in Section 5, Board (2011), Andrews and Barron (2016), and Barron and Powell (2018) study relational contracts in which parties cannot coordinate punishments. However, these papers assume that uncoordinated punishments arise due to private monitoring rather than collusion.

A growing body of literature studies how agency problems interact with financial constraints (e.g., Pagano and Vulpin (2008) and Matsa (2017)). Michelacci and Quadrini (2009) argues that firms might defer wage payments to “borrow money” from workers, but unlike our model, that paper assumes commitment and does not focus on productivity dynamics. Li and Matouschek (2013), Englmaier and Fahn (2017), and Fuchs et al. (2017) consider liquidity constraints or investments in relational contracts, but these papers do not study how borrowing constrains incentives.

Complementing this theoretical literature, a number of papers have explored the empirical relationship between financial decisions and labor considerations. Brown and Matsa (2016) shows that firms in financial distress are less able to attract job applicants, consistent with our result that indebted firms are less able to promise attractive rewards. Graham et al. (2019) studies the relationship between decreases in creditworthiness and increases in wage premia, which, they argue, compensate workers for the possibility of bankruptcy. Other papers explore related but distinct mechanisms that link debt to wages; Matsa (2010) argues that firms take on debt to improve their bargaining position versus organized labor, while Benmelech et al. (2012) finds that firms in financial distress negotiate substantially less generous labor contracts. Kale and Shahrur (2007) investigates how a firm’s capital structure affects its relationships with suppliers and customers. We complement these papers by studying an organizational reason why debt might constrain incentives, exploring the resulting debt and firm dynamics, and documenting new stylized facts that relate debt to worker morale. In contrast to many of these papers, which study firms in financial distress,

our model suggests that debt can have substantial effects on worker morale even if there is minimal risk of bankruptcy.

We analyze our model using the now-standard tools developed by Thomas and Worrall (1994), Albuquerque and Hopenhayn (2004), and DeMarzo and Fishman (2007), with effort serving the role of working capital. It is, perhaps, surprising that these techniques apply since, unlike these papers, our manager interacts with two parties (i.e., a creditor and a worker) rather than one. The key result that allows us to adapt these methods is our Lemma 1, which shows that the creditor and worker essentially act as a single player in equilibrium. This lemma does not hold if we allow collusion, as in Section 5, so the proofs in that section rely on a different set of techniques.

More broadly, we build on the vast literature that studies commitment problems in financial relationships (Jensen and Meckling (1976); Myers (1977); Townsend (1979); Gale and Hellwig (1985); Stein (2003)). In particular, Aghion and Bolton (1992), Hart (1995), Holmstrom and Tirole (1997), and others focus on commitment problems in financial contracts. More recently, He and Milbradt (2016), DeMarzo and He (2017), and the papers discussed in the previous paragraph consider dynamic models of financing with limited commitment. The point of this paper is to show that these commitment problems meaningfully interact with worker incentives. Our analysis of collusion relates to Tirole (1986), Biais and Gollier (1997), and other static models of collusion.

## 2 Debt and Morale: Stylized Facts

This section uses various firm- and worker-level data to motivate our theory. We document that a firm's leverage is *positively* related to the probability that it reports problems with workforce motivation and turnover and *negatively* related to employee self-reported job satisfaction and morale. We also show that increases in leverage are *negatively* related to changes in wages and total factor productivity, and this association is strongest in countries

with high costs of contract enforcement and in industries that rely heavily on labor.

We begin with the relationship between debt and employee morale. Studying this relationship requires firm-level data about morale, which are not available in most datasets.<sup>3</sup> We use firm- and worker-level surveys administered by the German Employment Agency, which contain information on firms’ financials and wages, as well as workers’ and firms’ assessments of employee morale.

For workers’ assessments of morale, we use the 2012 and 2014 waves of the Linked Personnel Panel (LPP), which contain a series of questions about employees’ attitudes towards their jobs. For firms’ assessments of morale, we use the responses from the IAB Establishment Panel (IAB-EP), which is conducted by the German Employment Agency and contains information about firm financials and perceived personnel problems. Due to data availability, we focus on 2004–2008 in the IAB-EP data. Not every variable is available in every year. We measure debt by the share of loans in investment volume, which is available for 2005 and 2008, while firms’ assessments of workforce motivation and turnover are available for 2004, 2006, and 2008. Given this data structure, we present results from three specifications: (1) a cross-section of firms that averages all variables from IAB-EP data observed between 2004 and 2008; (2) a panel dataset with two observations per firm;<sup>4</sup> and (3) a cross-section of workers, pooling responses from the two waves of the LPP and merging them with the 2008 measure of firm leverage from IAB-EP. Appendix B discusses the data in more detail.

Table 1 documents firm-level regressions of morale on leverage. Columns (1)–(2) and (5) consider variations of the cross-sectional regression:

$$Morale_i = \beta_0 + \beta_1 Lev_i + \boldsymbol{\beta} \mathbf{X}_i + \epsilon_i, \tag{1}$$

where  $Morale_i$  is a measure of employee morale,  $Lev_i$  is a measure of leverage, and  $\mathbf{X}_i$  is a

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<sup>3</sup>There is a large body of literature in organizational behavior that studies the relationship between employee well-being and productivity (Krekel et al., 2019), but few papers study how firm’s actions affect morale, and the ones that do typically focus on compensation and benefits programs (Jones et al., 2018).

<sup>4</sup>We use the measure of leverage from 2005 and the measure of staffing problems from 2006 as the first observation. The second observation uses values from 2008 for both variables.

Table 1: Firm-level Debt, Workforce Motivation and Turnover

	CROSS-SECTION		PANEL		CROSS-SECTION	PANEL
	PROBLEMS WITH MOTIVATION				PROBLEMS WITH TURNOVER	
	(1)	(2)	(3)	(4)	(5)	(6)
Leverage	0.047*** (0.007)	0.034*** (0.008)	0.020+ (0.014)		0.011** (0.005)	0.0236** (0.010)
Leverage>0				0.0226*** (0.0088)		
N	14 124	9 677	17 522	17 522	9 677	17 522
Fixed Effects	Industry		Firm		Industry	Firm
Add. Controls	-	✓	✓	✓	✓	✓

The table shows the regression of firms' declarations about problems with the workforce on measures of debt. Columns 1, 2, and 5 show cross-sectional regressions using averages from the 2004–2008 period. Columns 3, 4 and 6 use panel data with two observations per firm. The dependent variable in columns 1-4 is a binary indicator for “problems with workforce motivation”, as declared by the firm. The dependent variable in columns 5-6 is a similar indicator for “problems with workforce turnover”. The questions we use to measure staffing problems were asked in 2004, 2006, and 2008. Our cross-sectional columns use averages of these 3 values, while our panel waves use values from 2006 and 2008. The independent variable in the first row is share of debt in the investment volume, which is available in 2005 and 2008. In the panel columns, these two values are used for two waves, while in the cross-sectional columns the average of these two values is used. The independent variable in the second row is a binary indicator for the average share of debt exceeding zero, where we define this variable for the panel and cross-sectional columns analogously to the share of debt. Our controls include either firm fixed effects or industry fixed effects (using the German Employment Agency classification, which roughly corresponds to 2 digit SIC codes) and indicators for five quintiles of total employment. Our additional controls include five indicators for each firm's self-reported profitability, its type of management, the growth in its employment, and its average wage. In the panel columns, we add year fixed effects and drop management type, because it is absorbed by firm fixed effects. Standard errors are shown in parentheses. In the panel regression columns, standard errors are clustered on the firm level. (\*\*\*) denotes significance at 1% level, (\*\*) at 5%, (\*) at 10% and (+) at 15% level.

vector of controls that include industry fixed effects, firm size quintiles, and a self-reported measure of firm profitability. Thus, this regression controls for the firm's own perceptions of its profits. The unit of observation  $i$  is a firm, where we average responses across years. Columns (3)–(4) and (6) give results for the panel regression:

$$Morale_{it} = \beta_0 + \beta_1 Lev_{it} + \beta \mathbf{X}_{it} + \xi_i + \epsilon_{it}, \quad (2)$$

where  $\xi_i$  represents firm fixed effects. We include two waves,  $t \in \{2005/2006, 2008\}$ , where 2005/2006 means the leverage measure from 2005 and the morale measure from 2006.

The first four columns of Table 1 illustrate a positive firm-level relationship between leverage and motivational problems. For example, column (2) says that one standard deviation increase in leverage is associated with a 1% increase in the probability that a firm



reports problems with motivating employees, which constitutes 12% of the base probability. The coefficients in our cross-sectional regression, (1)–(2), are highly significant, while the coefficient in (3) is marginal (significant at 15%). Given that our panel is very short, column (4) pools our leverage measure into an indicator that equals one if the firm has any debt. This binary measure of leverage is significantly (at 1%) and positively related to the probability that the firm reports problems with motivating workers. Columns (5) and (6) give the corresponding cross-sectional and panel regressions when the dependent variable is an indicator that equals one if the firm reports problems with workforce turnover. Both the cross section and the panel regression exhibit a significant, positive relationship (at 5%) between leverage and problems with worker retention.

We complement these results with data on employees’ assessments of their own motivation. Table 2 considers the following specification:

$$Morale_{j,i} = \beta_0 + \beta_1 Lev_i + \boldsymbol{\beta} \mathbf{X}_{j,i} + \epsilon_j, \quad (3)$$

where  $i$  denotes a firm and  $j$  denotes a worker. In the basic specification, the vector  $\mathbf{X}_{j,i}$  includes the worker’s sex and age, while the specification with additional controls includes the worker’s education, income, type of contract with the firm, and the firm’s profitability and average wage. Columns (1) and (2) show that workers at more highly leveraged firms report that they think more frequently about leaving their jobs. These results are highly significant (at 1%) both without and with controls. Columns (3) and (4) repeat this exercise using workers’ average responses to six categorical questions regarding their commitment to the firm. Higher leverage is correlated with workers reporting that they are less committed to the firm, although this relationship is marginal (significant at 15%) once we include controls. Columns (5) and (6) document a correlation between leverage and job satisfaction, although again, this correlation is only marginally significant (at 10%) after controls. Overall, these regressions provide evidence that is consistent with the firm-level correlation between leverage and morale documented in Table 1.

Table 2: Firm-level Debt and Worker-level Motivation

	CONSIDERS QUITTING	COMMITMENT TO FIRM	JOB SATISFACTION			
	(1)	(2)	(3)	(4)	(5)	(6)
Leverage	0.227*** (0.069)	0.201*** (0.066)	-0.157** (0.070)	-0.113+ (0.070)	-0.244** (0.113)	-0.213* (0.113)
N	7 637	7 399	7 647	7 408	7 645	7 407
Add. Controls		✓		✓		✓

The table shows the results of the regression of workers' assessments of their morale on measures of their employer's debt. All our regressions are cross-sectional, use the average of workers' answers in waves 2012 and 2014 of the LPP data, and merge those answers with the 2008 measure of the firms' share of debt in investment expenditures from the IAB-EP data. The dependent variable in columns 1-2 is a response to the question "How often have you thought about changing your job in past 12 months?" on a 5-point scale: never (1), a few times a year (2), a few times a month (3), every week (4), and every day (5). The dependent variable in columns 3-4 is an average of six prompts that measure a worker's commitment to the firm on a 5-point scale. The prompts are: I am happy to spend rest of my life with the firm; the firm has a great deal of personal meaning for me; the firm's problems are my own; I feel a sense of belonging; I feel emotionally attached to the firm; and I feel part of the family at my firm. The dependent variable in columns 5-6 measures each employee's job satisfaction on 10-point scale. The independent variable is the share of debt in the firm's investment volume in 2008. Each column controls for worker's age and sex. Our additional controls include the worker's educational and income levels, the average wage at the firm, the type of the employee's contract (fixed term or permanent), and five self-reported indicators of firm profitability. Our standard errors are shown in parentheses and clustered on the firm level. (\*\*\*) denotes significance at 1% level, (\*\*) at 5%, (\*) at 10% and (+) at 15% level.

We argue that leverage affects not only morale but also worker pay and firm productivity. To explore these links, we use data from Amadeus, which does not have information about employee morale but includes much more detailed financial data, a larger sample of European firms, and a longer panel. Our sample covers 16 countries and almost 25,000 public and private manufacturing firms, with up to 10 years of data per firm between 1985–2017.

We measure borrowing by the ratio of non-current liabilities to total assets (book leverage); unlike market leverage, this measure of debt does not directly depend on market beliefs about current or future profits. We proxy for productivity using revenue total factor productivity (TFP-R), which we calculate using a method from Gandhi et al. (2017) that is designed to control for the endogenous choice of inputs. We use output revenue, fixed assets, the total wage bill, and the cost of materials to proxy for output, capital, labor, and intermediate inputs, respectively. Our analysis restricts attention to industry-country pairs with at least 1,000 observations; Appendix B provides additional details about the data.

Table 3 presents the results of this analysis. Columns (1) and (2) regress changes in wages on changes in both current- and previous-year leverage:

$$\Delta Wage_{it} = \beta_1 \Delta Lev_{it} + \beta_2 \Delta Lev_{i(t-1)} + \beta_3 X_{it} + \mu_i + \psi_t + \epsilon_{i,t}. \quad (4)$$

Here,  $\Delta Wage_{it}$  is the wage change from year  $t-1$  to year  $t$ , where wages are measured as total compensation costs divided by the number of workers. Similarly,  $\Delta Lev_{i,t}$  is the change in financial leverage from  $t-1$  to  $t$ . We include both firm and year fixed effects, so our regression exploits within-firm changes in productivity and leverage while controlling for year-specific aggregate shocks. We also run an alternative specification that includes industry-year fixed effects in order to control for year-specific shocks to industry-level productivity. Additional controls,  $X_{i,t}$ , include changes in the number of employees and total fixed assets. Our main results weight observations equally; we find similar results if we weight by the number of employees, total assets, or gross output, though  $\Delta Lev_{i,t-1}$  is not always significant. Standard errors are clustered by NACE2 industry code.

We find that an increase in leverage is correlated with a decrease in contemporaneous wages, a correlation that is highly significant (at 1%). Lagged leverage changes are typically negatively correlated with wages, although this effect is only marginally significant.

Columns (5)-(6) repeat these exercises with our measure of firm productivity. Increases in firm leverage are correlated with decreases in productivity. This relationship is highly significant and economically substantial: one standard deviation increase in contemporary leverage is correlated with an decrease in TFP-R equal to 11.5–13.4% of the median within-firm standard deviation.<sup>5</sup> Interestingly, *lagged* increases in leverage are also correlated with decreases in productivity, which suggests that this association is persistent.

The central friction in this model is a commitment problem: the manager can renege on payments to workers and repayments to creditors. This commitment problem is plausibly more severe, so the relationship between debt, compensation, and productivity is stronger, in countries with weak formal contracting institutions. To investigate this claim, Columns

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<sup>5</sup>The standard deviation of log-change in leverage in our final sample is 0.044, while the within-firm standard deviation of TFP changes for the median firm equals 0.156.

Table 3: Firm Debt, Wages and Productivity

	$\Delta L(\text{Wage})$				$\Delta \text{TFP}$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\Delta \text{Leverage}$	-0.109*** (0.0144)	-0.0832*** (0.0164)	0.0016 (0.0207)	-0.0366** (0.0165)	-0.475*** (0.075)	-0.409*** (0.065)	-0.292*** (0.042)	-0.290*** (0.0612)
$\Delta \text{Leverage (t-1)}$	-0.0180+ (0.0125)	-0.0183* (0.0104)	-0.0468*** (0.0161)	-0.0498*** (0.0141)	-0.090* (0.021)	-0.065*** (0.011)	-0.118*** (0.034)	-0.0884*** (0.0196)
$\Delta \text{Leverage} \times \text{Costly Enforcement}$			-0.186*** (0.036)				-0.332*** (0.098)	
$\Delta \text{Leverage (t-1)} \times \text{Costly Enforcement}$			0.0017 (0.0151)				0.046 (0.036)	
$\Delta \text{Leverage} \times \text{Labor Intensive}$				-0.110*** (0.0274)				-0.313*** (0.108)
$\Delta \text{Leverage (t-1)} \times \text{Labor Intensive}$				0.0085 (0.0211)				-0.00601 (0.0390)
N	127703	127703	127604	127703	127703	127703	127604	127703
Firm FE	✓		✓	✓	✓		✓	✓
Year FE	✓		✓	✓	✓		✓	✓
Industry X Year FE		✓				✓		

The table shows the regression of wages and productivity on financial leverage using firm-level data from Amadeus. The dependent variable in columns 1–4 is the change in the log of average wages, which is calculated as the ratio of total compensation costs to the number of employees. The dependent variable in columns 5–8 is the change in TFP-R, which is calculated using method developed by Gandhi et al. (2017). We measure Leverage as the ratio of non-current liabilities to total assets (book value). The variable 'Costly Enforcement' is an indicator that equals 1 if the country-level measure of cost of contract enforcement (from the Doing Business Survey) is above the median, and equals 0 otherwise. The variable 'Labor Intensive' is an indicator that equals 1 if the industry share of labor input in its OLS-based production function is above the country-wide median, and equals 0 otherwise. Each column includes total fixed assets and number of employees as additional controls. We cluster standard errors at the industry level. These standard errors are displayed in the parentheses. (\*\*\*) denotes significance at 1% level, (\*\*) at 5%, (\*) at 10% and (+) at 15% level.

(3) and (7) run a version of Equation (4) that includes interactions between current and lagged leverage and a country-level measure of the cost of contract enforcement from the World Bank’s Doing Business survey.<sup>6</sup> This measure is used by Fahn et al. (2017) as a proxy for the prevalence of commitment problems in incentive contracts. We find that leverage increases are more strongly correlated with wage and productivity decreases in countries with costlier formal enforcement.

Our proposed mechanism operates through a labor channel, suggesting that leverage should also have a more pronounced effect on wages in industries that rely heavily on labor inputs. Columns (4) and (8) investigate this source of heterogeneity by interacting leverage with a country-industry level proxy for labor intensity, as measured by an indicator that equals 1 whenever the OLS-based coefficient of labor in the production function for a given industry is above the country-wide median.<sup>7</sup> This interaction term is negative and significant, suggesting that the association between leverage, compensation, and productivity is stronger in more labor-intensive industries.

Overall, our results suggest that debt is negatively related to employees’ morale, wages, and productivity. Of course, these relationships should be interpreted with caution, because they document correlations rather than causal relationships. Nevertheless, they suggest a possible relationship between a firm’s financial obligations and its ability to motivate workers. The remainder of the paper presents a dynamic model of firm-worker relationships that is consistent with these observations.

### 3 Model

Consider an infinite-horizon game with a manager (‘she’), a creditor (‘it’), and a worker (‘he’) who share a common discount factor  $\delta \in [0, 1)$ . The manager needs a loan of size  $L > 0$

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<sup>6</sup>Our main results use a binary indicator of an above-median cost of contract enforcement, but the analysis is robust to using a continuous measure instead.

<sup>7</sup>We do not need to include this indicator as a separate regressor because it is absorbed by firm fixed effects.

to start a project. She writes a contract to borrow money from a deep-pocketed creditor. The contract specifies a liquidation probability in each period,  $l(\cdot) \in [0, 1]$ , contingent on the history of payments to the creditor. The payment to the creditor in period  $t \in \{0, 1, \dots\}$  is denoted  $r_t$ , with corresponding contractible history  $h_c^t = (r_0, \dots, r_t)$ .

The creditor accepts or rejects this contract. If it accepts, then it pays  $L$ , the project is funded, and the game continues; otherwise, the game ends and players earn 0. This decision is the creditor's only action in the game.

If the project is funded, then the manager and the worker interact repeatedly. All actions are publicly observed, but the only contractible variables are repayments to the creditor ( $r_t$ ) and a public randomization device that is realized at every stage of each period. In each period  $t \in \{0, 1, \dots\}$ , the manager and worker play the following stage game:

1. The worker chooses effort  $a_t \in \mathbb{R}_+$ .
2. A state of the world  $\theta_t \in \{0, 1\}$  is realized, with  $\Pr\{\theta_t = 1\} = p$ .
3. Output  $y_t = \theta_t a_t$  is realized.
4. The manager pays  $b_t \geq 0$  and  $r_t \geq 0$  to the worker and creditor, respectively, where
 
$$b_t + r_t \leq y_t.$$
5. The project is liquidated with probability  $l_t \equiv l(h_c^t)$ .

If the project has not yet been liquidated, then the manager's and worker's period- $t$  payoffs are  $\pi_t = y_t - b_t - r_t$  and  $u_t = b_t - c(a_t)$ , respectively, where  $c(\cdot)$  is non-negative, strictly increasing, strictly convex, and satisfies  $c(0) = c'(0) = 0$  and  $\lim_{a \rightarrow \infty} c'(a) = \infty$ . The creditor earns  $r_t$  in period  $t$ . Following liquidation, the game ends and all players earn 0. The manager's, worker's, and creditor's normalized discounted continuation payoffs in period  $t$  are  $\Pi_t = \sum_{t'=t}^{\infty} \delta^{t'-t}(1 - \delta)\pi_{t'}$ ,  $U_t = \sum_{t'=t}^{\infty} \delta^{t'-t}(1 - \delta)u_{t'}$ , and  $K_t = \sum_{t'=t}^{\infty} \delta^{t'-t}(1 - \delta)r_{t'}$ , respectively.

Let **first-best effort**  $a^{FB}$  satisfy  $c'(a^{FB}) = p$ . We assume throughout that  $pa^{FB} - c(a^{FB}) > L$  so that the project has strictly positive net-present value if the worker exerts first-best effort. Define  $h^t \in \mathcal{H}^t$  as a history in period  $t$ . We consider **profit-maximizing** subgame-perfect equilibria (SPE), which maximize the manager's *ex ante* expected payoff among SPE.

It is worth spending a moment on the central contracting friction in this model, which is that the manager can refuse to pay  $r_t$  or  $b_t$  and suffer no worse punishment than losing her future returns from the project. Consistent with the literature on relational contracts, we model this situation by assuming that the formal contract cannot depend on  $y_t$ , which means that the manager can always renege on promised payments. While financial instruments do typically depend on realized profits, managers also typically face opportunities to divert those profits in non-contractible ways, for instance, by stealing them directly or diverting them to privately beneficial but unprofitable ventures (see, e.g., Hart (1995); Holmstrom and Tirole (1997); Albuquerque and Hopenhayn (2004); and DeMarzo and Fishman (2007)). Our formulation is equivalent to one in which *reported* profits are contractible, but the manager can mis-report profits and pocket the difference.<sup>8</sup>

Several features of the model are convenient but inessential for our basic intuition. For instance, we assume the manager keeps the entire output  $y_t$  if she reneges on equilibrium payments, but our intuition would continue to hold if she keeps only a (strictly positive) fraction of output after reneging. Further, players earn 0 upon liquidation, but allowing a positive scrap value would not substantively change the analysis. We assume that the creditor observes everything in the repeated game, but this assumption is irrelevant because the creditor takes no actions after it accepts the initial contract.

Other assumptions potentially affect the central intuition. First, apart from the initial loan, the manager can neither borrow nor save; we discuss this restriction further in Section

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<sup>8</sup>For example, we could assume that reported profit  $\hat{y}_t$  is contractible, but the manager can report  $\hat{y}_t < y_t$  and effectively steal  $y_t - \hat{y}_t$ . If payments depend on  $\hat{y}_t$  but not  $y_t$  and, moreover, must satisfy  $b_t + r_t \leq \hat{y}_t$ , then our analysis would continue to hold.

6. Second, effort is observable, which simplifies incentives to starkly highlight how financing constraints lead to effort dynamics. Finally, we assume that liquidation  $l(\cdot)$  can depend on repayments to the creditor but not on payments to the worker, which is irrelevant in Section 4 but important in Section 5, because it allows the manager and worker to “secretly” make side payments to one another while colluding.

## 4 Financial Constraints and Morale

This section analyzes productivity and payment dynamics in profit-maximizing equilibria. To do so, we formulate the equilibrium payoff frontier as a dynamic program, consider two benchmarks that highlight the role of the manager’s commitment problem, and then prove our main results. All proofs are included in Appendix A.

The worker or the creditor can punish the manager equally harshly following a deviation, the former by choosing  $a_t = 0$  in each period and the latter by liquidating the firm. While either of these punishments min-max the manager, only the worker can condition his punishment on realized output,  $y_t$ . Consequently, it is efficient for the worker to punish *any* deviation by the manager, including deviations in  $r_t$ . We can, therefore, set  $l(\cdot) \equiv 0$  and assume that the manager earns 0 following any deviation.

Given these punishments, we can write the dynamic program for a profit-maximizing equilibrium. Let

$$E = \{(U, \Pi) | \exists K \in \mathbb{R} \text{ s.t. } \exists \text{ SPE with worker, manager, and creditor payoffs } (U, \Pi, K)\}$$

be the set of the worker’s and the manager’s equilibrium continuation payoffs. Given  $(U, \Pi) \in E$ , define  $K(U, \Pi)$  as the creditor’s maximum continuation payoff if the worker and manager earn  $U$  and  $\Pi$ , respectively. If  $\theta_t = 1$  in period  $t \geq 0$ , let  $b_t \equiv b \geq 0$  and  $r_t \equiv r \geq 0$  be the manager’s payments to the worker and creditor and note that  $b_t = r_t = 0$  if  $\theta_t = 0$ . Let  $(U_L, \Pi_L)$  and  $(U_H, \Pi_H)$  be the worker’s and manager’s continuation surpluses from period



$t + 1$  onwards if  $\theta_t = 0$  or  $\theta_t = 1$ , respectively.

Equilibrium strategies must satisfy several sets of constraints. Consider period  $t \geq 0$  of an equilibrium in which the manager and worker earn  $\Pi$  and  $U$ , respectively. The **promise-keeping constraints** require that the parties earn these payoffs:

$$U = (1 - \delta)(pb - c(a)) + \delta(pU_H + (1 - p)U_L) \text{ and} \quad (\text{PK-A})$$

$$\Pi = (1 - \delta)p(a - b - r) + \delta(p\Pi_H + (1 - p)\Pi_L). \quad (\text{PK-P})$$

The **incentive constraint** ensures that the worker is willing to choose the equilibrium effort  $a \geq 0$ . Since he earns  $U$  from doing so and no more than 0 from deviating, this constraint is simply

$$U \geq 0. \quad (\text{IC})$$

The **dynamic enforcement constraint** captures the manager's commitment problem. If  $\theta = 1$ , she must prefer to pay  $b$  and  $r$  rather than pay nothing and earn 0 continuation payoff:

$$\delta\Pi_H \geq (1 - \delta)(b + r). \quad (\text{DE})$$

**Limited liability constraints** require  $r$  and  $b$  to be no more than realized output:

$$r \geq 0; \quad b \geq 0; \quad r + b \leq a. \quad (\text{LL})$$

Finally, continuation payoffs must be feasible in equilibrium, so

$$(U_H, \Pi_H), (U_L, \Pi_L) \in E. \quad (\text{CE})$$

Given  $(U, \Pi)$ , the equilibrium that maximizes the creditor's payoff solves

$$K(U, \Pi) \equiv \max_{r, b, a, U_H, U_L, \Pi_H, \Pi_L} (1 - \delta)pr + \delta (pK(U_H, \Pi_H) + (1 - p)K(U_L, \Pi_L)) \quad (5)$$

subject to (PK-A), (PK-P), (IC), (DE), (LL), (CE).

The creditor is willing to loan the manager funds as long as at the start of the game, continuation utilities,  $(U_0, \Pi_0) \in E$ , satisfy  $K(U_0, \Pi_0) \geq L$ . Therefore, characterizing equilibrium dynamics amounts to characterizing the equilibrium payoff frontier  $K(\cdot, \cdot)$ .

Before turning to this characterization, we consider two benchmarks. The first benchmark eliminates the manager's commitment problem, which removes (DE) as a constraint. In the **commitment game**, the formal contract specifies payments  $b_t$  and  $r_t$  as a function of the history of realized outputs,  $\{y_{t'}\}_{t'=0}^t$ . These payments do not need to satisfy (DE), because the manager cannot renege on them. We show that profit-maximizing equilibria attain first-best in this benchmark.

**Proposition 1** *In any profit-maximizing equilibrium of the commitment game, the project is funded and  $a_t = a^{FB}$  for all  $t \geq 0$ .*

With output-contingent formal contracts, the manager can commit to repay the creditor and pay the worker for his costs. She is then the residual claimant on the (positive net-present value) project, so she optimally induces first-best effort in each period.

Our second benchmark removes (IC) as a constraint by analyzing a setting in which the worker is essentially passive. In the **bilateral game**, the worker chooses  $a_t$ , which the manager can either accept or reject. If she rejects, then effort and output equal 0 in that period; if she accepts, then effort equals  $a_t$  and the manager—but *not* the worker—incurs cost  $c(a_t)$ . This benchmark essentially eliminates the need to motivate the worker, so we can set  $b_t = 0$  in each period. As in the baseline game, however, the worker can still punish the manager following a deviation by choosing  $a_t = 0$  in subsequent periods.

Because the manager cannot commit to  $r_t$ , she must earn a strictly positive payoff or else she will renege on repaying the creditor. This commitment problem drives a wedge between the value created by the project and the value that can be pledged to the creditor. Conditional on the project being funded, however, the manager can implement any effort, so  $a_t = a^{FB}$  in each period.

**Proposition 2** *In any profit-maximizing equilibrium of the bilateral game, the project is funded if and only if  $L \leq \frac{\delta p}{1-\delta+\delta p} (pa^{FB} - c(a^{FB}))$ . If the project is funded, then  $a_t = a^{FB}$  for all  $t \geq 0$ .*

We can set  $b_t = 0$  in each  $t \geq 0$  in the bilateral game, in which case the manager earns  $pa_t - c(a_t) - r_t$ . Setting  $a_t = a^{FB}$  both maximizes the manager's payoff and the amount she stands to lose from renegeing on the creditor. However, the manager must earn a strictly positive payoff to be willing to repay the loan, which means that she can pledge only a fraction of the project's proceeds to the creditor.

In both of these benchmarks, profit-maximizing equilibria entail stationary (and first-best) effort. We now show that profit-maximizing equilibria in the full game are necessarily dynamic. To do so, we first simplify the dynamic programming problem by proving that the manager's maximum payoff depends on the aggregate surplus promised to the creditor and the worker,  $K(U, \Pi) + U$ , but not on how that surplus is split between these parties. Define

$$\tilde{U}(\Pi) = \max \{U | (U, \Pi) \in E\}$$

as the worker's maximum equilibrium continuation utility given  $\Pi$ .

**Lemma 1** *For any  $(U, \Pi) \in E$ ,*

$$K(U, \Pi) + U = \tilde{U}(\Pi).$$

Lemma 1 says that the manager's equilibrium payoff depends only on the *sum* of the creditor's and worker's payoffs. This result substantially simplifies our analysis, because it means that we only need to track the manager's aggregate obligations to the creditor and worker in equilibrium. To prove Lemma 1, we show that for any  $(U, \Pi)$  satisfying  $K(U, \Pi) + U > 0$ , we can find a period in which we can perturb the equilibrium to transfer utility between the creditor and the worker without violating any equilibrium constraints. Consequently, we can vary  $U \geq 0$  while holding  $K(U, \Pi) + U$  fixed. This argument follows from the fact that, as in Levin (2002), the manager is held to her min-max payoff following any deviation. Unlike that paper, however, Lemma 1 holds even though the manager and the worker are both liquidity constrained, and it does *not* imply that profit-maximizing equilibria are stationary. Indeed, our next result shows that these equilibria typically entail dynamics.<sup>9</sup>

Given Lemma 1, we can characterize the equilibrium payoff frontier in terms of  $\tilde{U}(\Pi)$ . Define

$$a_{max} = \arg \max_{a \geq 0} \left\{ pa - c(a) \mid \delta(pa - c(a)) \geq \frac{(1 - \delta)c(a)}{p} \right\} \quad (6)$$

as the maximum effort that can be attained in equilibrium if  $K(U, \Pi) = 0$ . Our next result is the main result of this model and demonstrates how debt affects morale in profit-maximizing equilibria.

**Proposition 3** *Consider on-path play in period  $t$  of a profit-maximizing equilibrium, and let  $\Pi$  be the manager's continuation payoff at the start of that period. Then,*

1.  $a_t = a^*(\Pi)$ , where

$$a^*(\Pi) \equiv \min \left\{ a_{max}, \frac{(1 - \delta(1 - p))\Pi}{(1 - \delta)p} \right\}.$$

2. If  $a_t < a_{max}$ , then  $\Pi_L = \Pi < \Pi_H$ ,  $a_{t+1} = a_t$  if  $\theta_t = 0$ , and  $a_{t+1} > a_t$  if  $\theta_t = 1$ .

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<sup>9</sup>We can show that a version of Lemma 1 holds in more general settings with multiple workers and creditors if effort is observable and players jointly punish deviations. Consequently, similar productivity dynamics arise in richer environments.

3. If  $a_t = a_{max}$ , then  $a_{t'} = a_{max}$  for all  $t' > t$ . Consequently,

$$\lim_{t \rightarrow \infty} Pr\{a_t = a_{max}\} = 1.$$

Whenever the project is funded in the profit-maximizing equilibrium, the agent's initial continuation payoff equals  $U_0 = 0$ , and the manager's payoff  $\Pi_0$  satisfies  $K(0, \Pi_0) = L$ . Proposition 3 implies that from this starting point, effort remains constant after  $\theta_t = 0$  and increases after  $\theta_t = 1$  until it reaches the steady state  $a_{max}$ .

To prove this result, recall that by Lemma 1, we only need to characterize  $\tilde{U}(\Pi)$ . In particular, we can set  $K = 0$  and consider equilibria that maximize the worker's payoff for any fixed manager payoff  $\Pi$ . Among those equilibria, we show that either  $a = a^{FB}$  or (DE) binds since, otherwise, the manager could profitably pay the worker more in exchange for higher effort. Therefore, (DE) pins down  $b$ ; in particular, if  $\tilde{U}(\Pi) = 0$ , then  $b = c(a)/p$  and  $a = a_{max}$ . If  $\tilde{U}(\Pi) > 0$ , then (IC) is slack, so we could increase  $a$  without violating it. However, the total surplus  $\tilde{U}(\Pi) + \Pi$  must be decreasing in  $\tilde{U}(\Pi)$ , since higher effort is more efficient (for  $a \leq a^{FB}$ ). Thus, the manager prefers front-loading the agent's pay as much as possible whenever  $\tilde{U}(\Pi_H) > 0$ .

The preceding argument is enough to pin down  $b$  and  $\Pi_H$ . Completing our characterization of  $\tilde{U}(\Pi)$  requires that we pin down  $\Pi_L$ , but  $\Pi_L$  and  $U_L$  matter only for the constraints (PK-A) and (PK-P). Therefore, the concavity of  $\tilde{U}(\Pi)$  implies that  $\tilde{U}(\Pi_L) + \Pi_L = \tilde{U}(\Pi) + \Pi$  in any profit-maximizing equilibrium. Thus,  $\Pi_L = \Pi$ , completing the characterization of  $\tilde{U}(\Pi)$ . Lemma 1 then implies that  $K(U, \Pi) = \tilde{U}(\Pi) - U$ .

For our purposes, the most important implication of Proposition 3 is that debt, as measured by  $K(U, \Pi)$ , has a negative effect on effort. Unless  $L$  (and hence  $K(0, \Pi_0)$ ) is small, effort is initially strictly below  $a_{max}$  and only slowly and stochastically increases to this steady state. That is, the manager's outstanding obligations to the creditor and the worker determine equilibrium effort. These outstanding obligations change over time with corresponding

changes in the worker's effort and pay.

While we focus on effort dynamics, profit-maximizing equilibria also exhibit two other inefficiencies. First, as in many relational contracting models (e.g., Levin (2003)), even steady-state effort,  $a_{max}$ , may be strictly less than  $a^{FB}$ . Second, as in our Proposition 2 and many papers on financing constraints (e.g., Holmstrom and Tirole (1997)), the project might not be funded even though it has a strictly positive net-present value.

We now turn to compensation dynamics. The key insight here is that while  $K(U_t, \Pi_t) + U_t$  is pinned down in every profit-maximizing equilibrium, the manager's individual obligations to the creditor and worker,  $K(U_t, \Pi_t)$  and  $U_t$ , are not. Different combinations of these individual obligations correspond to different equilibrium payment paths. Our next result identifies two extreme payment paths that repay the creditor either as quickly or as slowly as possible.

**Corollary 1** *The following payment paths are each part of a profit-maximizing equilibrium.*

1. The fastest repayment equilibrium:  $r_t = y_t$  whenever  $K(U_H, \Pi_H) > 0$ . Once  $K(U, \Pi) = 0$ ,  $b_t = y_t$  whenever  $U_H > 0$ .
2. The slowest repayment equilibrium: whenever  $K(0, \Pi_H) > 0$ , we have  $r_t = y_t - \frac{c(a_t)}{p}$  and  $b_t = \frac{c(a_t)}{p}$ . Thus,  $U_t = 0$  in every period.

Corollary 1 follows from Lemma 1 and the proof of Proposition 3. Before the agent's effort reaches its steady state, every profit-maximizing equilibrium entails the same *total* payment to the creditor and worker; the only question is how this payment is split between the two parties. To repay the creditor as quickly as possible, the manager pays the worker nothing until after the loan has been completely repaid. In the slowest repayment path, by contrast, she exactly compensates the worker for his cost in each period.

One implication of this result is that productivity dynamics might temporarily persist even after the creditor has been repaid since  $K(U, \Pi)$  alone does not determine equilibrium

effort. In particular, if  $K(U, \Pi) = 0$  but  $U > 0$  and  $U_H > 0$ , as is sometimes the case in the fastest repayment equilibrium, then effort remains strictly below  $a_{max}$  for at least one period after the manager has repaid the creditor. This result resonates with our empirical finding that past leverage changes are associated with negative current productivity shocks, even conditional on current leverage changes.

One consequence of Lemma 1 is that, since the worker and the creditor essentially act as a single entity in profit-maximizing equilibria, we can modify techniques from Albuquerque and Hopenhayn (2004) to characterize equilibrium effort dynamics. Of course, the models tackle substantively different questions, since our results focus on the morale of workers within the firm rather than working capital investments. Corollary 1 illustrates a second difference: our manager's obligations to her creditor can affect her outstanding obligations to her worker, for instance, if she defers worker compensation. Consequently, current debt is not a sufficient statistic for current morale. The next section considers a setting in which Lemma 1 does not hold, giving rise to different dynamics, including the possibility of on-path liquidation.

## 5 Collusion Between the Manager and Worker

Section 4 assumes that the worker is willing to punish the manager following a deviation, even if she reneges only on the creditor. These types of indirect punishments are potentially susceptible to collusion, since the manager and creditor would jointly benefit from renegeing on the creditor and splitting the resulting proceeds among themselves. This section explores the possibility of manager-worker collusion. To deter collusion, the creditor can threaten to liquidate the project unless it is repaid promptly. An unlucky firm might then be inefficiently liquidated on the equilibrium path, further increasing the implicit cost of leverage.

Introducing collusion to our model entails both conceptual and technical challenges. The technical challenge is that, under any reasonable model of manager-worker collusion, the

worker and creditor would not act as a single player in equilibrium. That is, Lemma 1 no longer holds, so we have to separately track the manager’s obligations to the creditor and the worker. The conceptual challenge is that, given the manager’s lack of commitment, manager-worker collusion should be self-enforcing in the context of their ongoing relationship. That is, the outcome of collusion should be an equilibrium of the game given the contract with the creditor.

Given these challenges, we focus on a simple binary-effort version of the model:  $a_t \in \{0, y\}$ , where (with abuse of notation)  $c(0) = 0$  and  $c(y) = c$ . Since this simplification limits equilibrium effort dynamics, we focus on payment and liquidation dynamics instead.<sup>10</sup>

To understand our model of manager-worker collusion, consider the following thought experiment. Suppose that after the creditor and principal agree to a contract, the principal can freely choose *any* continuation equilibrium. She will never choose an equilibrium in which the worker punishes her for renegeing on  $r_t$ , since she could instead choose one in which she is not punished, and the parties instead split  $r_t$  in a way that benefits both of them at the expense of the creditor. Consequently, the creditor cannot rely on the worker to punish deviations in  $r_t$ ; it can guarantee repayment only by threatening liquidation.

Online Appendix C introduces an equilibrium refinement that formalizes this thought experiment. This appendix shows that an equilibrium is immune to collusion if, whenever output is produced, the worker and the manager would jointly prefer to pay  $r_t$  rather than pocketing it and acting as if no output had been produced. We say that an equilibrium satisfying this (sufficient) condition is a truth-telling equilibrium.

**Definition 1** *An SPE  $\sigma^*$  is a **truth-telling equilibrium** if, at every on-path history  $h^t$  immediately before  $\theta_t$  is realized,*

$$E_{\sigma^*} [-(1 - \delta)r_t + \delta (\Pi_{t+1} + U_{t+1}) | h^t, \theta_t = 1] \geq E_{\sigma^*} [\delta (\Pi_{t+1} + U_{t+1}) | h^t, \theta_t = 0]. \quad (7)$$

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<sup>10</sup>Our analysis extends to a setting with continuous effort. In a supplemental file, available online at <https://sites.google.com/site/danielbarronecon/>, we consider the case with continuous effort. As with binary effort, we show that the manager optimally backloads worker pay whenever collusion is a binding concern, leading to similar implications for the relationship between debt, liquidation, and pay.



Online Appendix C discusses truth-telling equilibria in more detail. One attractive feature of (7) is that it is similar to the truth-telling constraints imposed in bilateral models of working capital dynamics, such as Clementi and Hopenhayn (2006) and DeMarzo and Fishman (2007). The key difference is that (7) depends on the *sum* of the worker's and the manager's payoffs rather than the creditor's payoff alone, reflecting the fact that collusion entails a joint rather than individual deviation.

We can write (7) recursively as the following **truth-telling** constraint:

$$(1 - \delta)r \leq \delta(\Pi_H + U_H - \Pi_L - U_L). \quad (\text{TT})$$

Let  $K^T(U, \Pi)$  be the solution to (5) subject to (PK-A)-(CE) and (TT), given  $a_t \in \{0, y\}$ . In a profit-maximizing truth-telling equilibrium, on-path continuation payoffs always lie on  $K^T(\cdot)$ . Our next result gives a partial characterization of this equilibrium payoff frontier, focusing on the relationship between debt, pay, and liquidation. Proofs for this section are included in Appendix A.

**Proposition 4** *Suppose*

$$\frac{c}{p} \leq \frac{\delta}{1 - \delta}(py - c). \quad (8)$$

*Then,*

1. *Fix  $\Pi \geq \frac{p(1-\delta)}{1-(1-p)\delta}y$ . If  $U \in \mathbb{R}_+$  is such that  $K^T(U, \Pi) > 0$ , then for any  $U' > U$  such that  $(U', \Pi)$  can be attained in a truth-telling equilibrium,  $K^T(U, \Pi) + U < K^T(U', \Pi) + U'$ .*
2. *If  $K^T(U_H, \Pi_H) > 0$ ,  $b = 0$  and  $r = y$ .*
3. *If  $K^T(U, \Pi) > 0$ , then liquidation occurs with strictly positive probability in the continuation equilibrium.*

Condition (8) ensures that a manager with zero debt can induce the worker to exert effort in equilibrium. The first part of Proposition 4 confirms that Lemma 1 does not hold for

truth-telling equilibria. The reason is that, unlike payments to the worker, repayments to the creditor must satisfy (TT). One consequence of this asymmetry, highlighted by the second part of Proposition 4, is that the manager initially uses her entire output to repay the creditor. By doing so, the manager “escapes” the additional constraint (TT) as quickly as possible. The worker is paid nothing and is motivated only by the promise of future compensation during these periods. The final part of the result identifies the reason why satisfying (TT) and, hence, taking on debt, is so costly: this condition can be satisfied for  $r > 0$  only if the project is liquidated with positive probability in the future. In short, the creditor threatens liquidation to deter collusion. The manager minimizes the probability of liquidation by frontloading loan repayment and backloading worker compensation. Therefore, total surplus and the worker’s continuation payoff depend on current and past debts.

The proof of this result is somewhat involved because we cannot rely on Lemma 1. We first argue that we can set  $a = y$  so long as the project has not yet been liquidated, since whenever  $a = 0$ , it would be at least as efficient to instead liquidate the firm. For  $r > 0$ , (TT) holds only if total continuation surplus varies with the current shock, which implies that liquidation must occur with positive probability on the equilibrium path. The manager devotes all output to repaying the creditor to minimize the probability of liquidation, which implies that the worker’s pay is deferred until after the creditor has been repaid.

Proposition 4 says that *every* profit-maximizing truth-telling equilibrium entails deferred worker compensation, which strengthens Corollary 1’s result that *there exist* profit-maximizing equilibria with deferred compensation. Since the project is sometimes liquidated in equilibrium, play eventually reaches one of two steady states; either the loan is repaid and continuation surplus converges to  $\frac{\delta}{1-\delta}(py - c)$ , or the firm is liquidated for failing to repay its debts.

## 6 Discussion

This section concludes the paper by informally discussing several extensions .

**Saving and borrowing:** Apart from the initial loan, the manager neither borrows nor saves in our model. If the manager could borrow additional funds from the creditor, then she could pay the worker more, which would decrease the worker’s promised continuation utility at the cost of increasing the creditor’s promised continuation utility. Without collusion, Lemma 1 suggests the *sum* of these utilities determines productivity, so the manager would not benefit from further borrowing. With collusion, Proposition 4 suggests that further borrowing would increase the probability of liquidation and would, therefore, be strictly inefficient. This intuition suggests that the manager would not benefit from further opportunities to borrow.

The effects of savings are a little more complicated. If the manager can still access her savings after deviating, then we believe that not much would change, as she would still like to repay her obligations as quickly as possible and would not save in profit-maximizing equilibria. In contrast, if the creditor or worker could stop the manager from accessing savings following a deviation, then accumulated savings could essentially serve as a bond to guarantee the manager’s promises. Such bonds can mitigate commitment problems, so might alter equilibrium dynamics.

**Investments with variable scale:** In some settings, the manager can choose the scale of her investment, with larger-scale investments having higher up-front costs, as well as higher returns. Our model suggests two opposing equilibrium distortions in the optimal investment scale. First, borrowing decreases effort, which should push the manager to pursue smaller, less expensive projects. Second, investing in certain kinds of projects can ease commitment problems, particularly if a larger project implies that the manager has more to lose following a deviation (Klein and Leffler (1981); Halac (2015); Englmaier and Fahn

(2017)). We can construct examples in which either of these two forces dominate, leading to either “insufficient” or “excessive” equilibrium investment relative to first-best.

**Conclusion:** Borrowing influences the credibility of a firm’s promised incentives. We propose a model to explore the interaction between debt, incentives, and effort. We show that debt has a negative effect on both morale and pay, results that resonate with our stylized facts relating leverage to low employee morale, low compensation, and low productivity. More generally, our analysis illustrates how the organizational structure *within* the firm can significantly impact the cost of its financial obligations and how it chooses to respond to economic shocks and new opportunities. Our results suggest that these types of spillovers influence firm productivity, profitability, and growth.

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# A Omitted Proofs

## A.1 Proof of Proposition 1

In the commitment game, suppose the manager offers a contract with  $r_t(y_t) = \frac{L}{p}1\{y_t \geq \frac{L}{p}\}$ ,  $b_t(y_t) = \frac{c(a^{FB})}{p}1\{y_t = a^{FB}\}$ , and  $l_t = 0$  in each  $t \geq 0$ . The creditor earns  $pr_t(y_t) = L$  from this contract, while the worker earns  $p\frac{c(a^{FB})}{p} - c(a^{FB}) = 0$  from choosing  $a^{FB}$  and no more than 0 from choosing any other effort. Therefore, this contract induces the creditor to fund the project and the worker to choose  $a_t = a^{FB}$  in each period. The manager earns  $pa^{FB} - c(a^{FB}) - L > 0$ , which is the maximum attainable surplus in any equilibrium. Therefore, this contract is profit-maximizing, and any profit-maximizing contract must fund the project and induce  $a^{FB}$  in each period. ■

## A.2 Proof of Proposition 2

In the unitary firm game, consider the following strategy: the manager offers a contract with  $l_t = 0$  in each  $t$ , which the creditor accepts. In each  $t$  on the equilibrium path,  $a_t = a^{FB}$ , the manager accepts this effort,  $b_t = 0$ , and  $r_t = 1\{y_t = a^{FB}\}\frac{L}{p}$ . Following any deviation, the worker chooses  $a_t = 0$  and the manager pays  $b_t = r_t = 0$ .

If  $L \leq \frac{\delta p}{1-\delta+\delta p}(pa^{FB} - c(a^{FB}))$ , then this strategy is an equilibrium. The the manager earns  $pa^{FB} - c(a^{FB}) - L > 0$  and accepts  $a_t = a^{FB}$ . She is willing to pay  $r_t = \frac{L}{p}$  as long as

$$(1 - \delta)\frac{L}{p} \leq \delta(pa^{FB} - c(a^{FB}) - L)$$

which is implied by  $L \leq \frac{\delta p}{1-\delta+\delta p}(pa^{FB} - c(a^{FB}))$ . She is able to pay  $r_t = \frac{L}{p}$  because  $\frac{L}{p} \leq a^{FB} - \frac{c(a^{FB})}{p} < a^{FB}$ . The manager earns the maximum attainable profit  $pa^{FB} - c(a^{FB}) - L > 0$  in this equilibrium, so if  $L \leq \frac{\delta p}{1-\delta+\delta p}(pa^{FB} - c(a^{FB}))$ , then any profit-maximizing equilibrium entails a funded project and  $a_t = a^{FB}$  in each period  $t$  on-path.

In any equilibrium,  $\Pi_t \leq pa^{FB} - c(a^{FB}) - K_t$  because  $U_t \geq 0$ . Therefore, (DE) requires

that at any  $h^t$  immediately following  $\theta_t = 1$ ,

$$(1 - \delta)r_t \leq \delta (pa^{FB} - c(a^{FB}) - E[K_{t+1}|h^t]). \quad (9)$$

For each  $t \geq 0$ , define

$$\mathcal{H}(t) \equiv \{h^t | \theta_t = 1, \theta_{t'} = 0 \forall t' < t\}$$

as the set of histories such that  $\theta_t = 1$  for the first time in period  $t$ . Then the project is funded only if

$$\sum_{t=0}^{\infty} \delta^t (1-p)^t p E[(1-\delta)r_t + \delta K_{t+1} | \mathcal{H}(t)] \geq L.$$

Applying (9) to this expression yields

$$\delta p \sum_{t=0}^{\infty} \delta^t (1-p)^t S^{FB} \geq L$$

or  $\frac{\delta p}{1-\delta+\delta p} S^{FB} \geq L$ . We conclude that the project cannot be funded if  $L > \frac{\delta p}{1-\delta+\delta p} S^{FB}$ . ■

### A.3 Proof of Lemma 1

We first prove that  $K(\tilde{U}(\Pi), \Pi) = 0$  for any  $(U, \Pi) \in E$ . Towards contradiction, suppose  $K(\tilde{U}(\Pi), \Pi) > 0$  for some  $(U, \Pi) \in E$ . Then, there exists some future period in which  $r > 0$ . In this period, consider the following perturbation: decrease  $r$  to  $\tilde{r} = r - \epsilon$  and increase  $b$  to  $\tilde{b} = b + \epsilon$ . For sufficiently small  $\epsilon > 0$ , these perturbed payoffs continue to satisfy (DE) and (LL) in that and all previous periods because  $r > 0$  and  $r + b = \tilde{r} + \tilde{b}$ , while (IC) is relaxed in that and all previous periods. However, then  $U < \tilde{U}(\Pi)$  and we obtain a contradiction. Therefore,  $K(\tilde{U}(\Pi), \Pi) = 0$  for all  $(U, \Pi) \in E$ .

Next, we prove that  $K(U, \Pi) = \tilde{U}(\Pi) - U$ . If  $U = \tilde{U}(\Pi)$ , then this result holds by the previous argument. Note that if  $\tilde{U}(\Pi) = 0$ , then  $U = \tilde{U}(\Pi)$ . Suppose  $0 \leq U < \tilde{U}(\Pi)$ . Then, we claim that  $K(U, \Pi) > 0$ . To prove this, consider the equilibrium in which the worker and manager earn  $\tilde{U}(\Pi)$  and  $\Pi$ , respectively. We claim that there exists some period  $t$  and

history  $h^t$  at the start of that period such that (i)  $b_t > 0$  if demand is high in that period, (ii)  $E[U_t|h^t] > 0$ , and (iii)  $E[U_{t'}|h^{t'}] > 0$  for any  $h^{t'}$  that precedes  $h^t$ .

To prove this, for each  $\tau \geq 1$ , define  $\mathcal{H}^\tau$  as the set of histories such that  $E[U|h^\tau] = 0$ , but  $E[U|h^t] > 0$  for all  $h^t$  that precedes  $h^\tau$ . Define  $\mathcal{H}^\infty$ , with element  $h^\infty \in \mathcal{H}^\infty$ , as the set of infinite-horizon histories for which  $E[U|h^\tau] > 0$  for every  $h^\tau$  that precedes  $h^\infty$ . Then,  $\mathcal{H}^\tau \cap \mathcal{H}^{\tau'} = \emptyset$  and  $\bigcup_{\tau=0}^{\infty} \mathcal{H}^\tau = \mathcal{H}$ , so the worker's payoff can be written as follows:

$$U = \sum_{\tau=0}^{\infty} \Pr_{\sigma^*} \{ \mathcal{H}^\tau \} \left( E \left[ (1 - \delta) \sum_{t=0}^{\tau-1} \delta^t (b_t - c(a_t)) | \mathcal{H}^\tau \right] \right).$$

Since  $U > 0$  and  $c(a_t) \geq 0$ , it cannot be that  $b_t \equiv 0$  in this expression. That is, there must exist some  $\tau$ ,  $h^\tau \in \mathcal{H}^\tau$ , and  $h^t$  that precedes  $h^\tau$  such that  $b_t > 0$  with positive probability at  $h^t$ . By definition,  $E[U_t|h^t] > 0$  and  $E[U_{t'}|h^{t'}] > 0$  for any  $h^{t'}$  that precedes  $h^t$ .

Consider decreasing  $b_t > 0$  at this  $h^t$  and increasing  $r_t$  by the same amount. This perturbation satisfies (DE) and (LL) because  $b_t > 0$  and  $b_t + r_t$  is constant, while (IC) is slack at  $h^t$  and every predecessor history, so it continues to hold for a sufficiently small perturbation. Since  $K + U$  remains constant in this perturbation, which can be performed for any  $U > 0$ ,  $K(U, \Pi) > 0$  whenever  $0 < U < \tilde{U}(\Pi)$ . If  $U = 0$ , then  $K(U, \Pi) > 0$ , and we can decrease  $r$  in some period holding  $b + r$  fixed to transfer utility from the creditor to the worker. Therefore,  $K(U, \Pi) + U$  is constant in  $\Pi$ , and  $K(U, \Pi) = \tilde{U}(\Pi) - U$ . ■

## A.4 Proof of Proposition 3

By Lemma 1,  $\tilde{U}(\Pi)$  characterizes equilibrium payoffs. Note that the public randomization device implies that  $\tilde{U}(\Pi)$  is concave. Let  $\Pi_{max}$  be the maximum  $\Pi$  such that  $(U, \Pi) \in E$ , and note that when  $U = \tilde{U}(\Pi)$ ,  $r_t = 0$  in every period  $t$  on the equilibrium path.

**Part 1:  $\tilde{U}(\Pi) + \Pi$  is increasing in  $\Pi$ , and strictly so unless  $\tilde{U}(\Pi) + \Pi = S^{FB}$ .** First, we claim that  $\tilde{U}(\Pi) + \Pi$  is weakly increasing in  $\Pi$ .

For any  $\Pi$  such that  $\tilde{U}(\Pi) > 0$ , the proof of Lemma 1 implies that there exists some history  $h^t$  such that  $b_t > 0$  with positive probability at  $h^t$ ,  $E[U_t|h^t] > 0$ , and  $E[U_{t'}|h^{t'}] > 0$  for any  $h^{t'}$  that precedes  $h^t$ . Consider decreasing  $b_t$ . Both (DE) and (LL) are relaxed by this change, while (IC) continues to hold in every previous period because it was previously slack. So this perturbation is consistent with equilibrium, increases  $\Pi$ , decreases  $U$ , and holds total surplus fixed. Hence, total surplus  $\tilde{U}(\Pi) + \Pi$  must be weakly increasing in  $\Pi$ .

Since  $\tilde{U}(\cdot)$  is concave, there exists a (possibly corner)  $\bar{\Pi}$  such that  $\tilde{U}(\Pi) + \Pi$  is strictly increasing for  $\Pi < \bar{\Pi}$ . We argue that (DE) binds whenever  $a < a^{FB}$ . If not, then we can perturb the equilibrium by increasing  $b$  and  $a$  so that  $pb - c(a)$  remains constant. This perturbation satisfies (IC) by construction, (DE) by assumption, and (LL) because  $c'(a) < p$  for  $a < a^{FB}$ .

Now, consider two cases. First, suppose

$$(1 - \delta) \frac{c(a^{FB})}{p} \leq \delta(pa^{FB} - c(a^{FB})). \quad (10)$$

Then, we claim that  $\tilde{U}(\bar{\Pi}) + \bar{\Pi} = S^{FB}$ . Consider the stationary strategy profile that sets  $a = a^{FB}$  and  $b = \frac{c(a^{FB})}{p}$ . This strategy profile clearly satisfies (IC) and (LL). It also satisfies (DE) because (10) holds. Therefore,  $(0, S^{FB}) \in E$  and, consequently,  $\tilde{U}(\bar{\Pi}) + \bar{\Pi} = S^{FB}$ .

Instead, suppose that (10) does not hold. Then, we claim that  $\tilde{U}(\bar{\Pi}) = 0$ . Suppose towards contradiction that  $\tilde{U}(\bar{\Pi}) > 0$ . If  $\Pi_L < \bar{\Pi}$ , then we can perturb the equilibrium by increasing  $\Pi_L$ , which would strictly increase total surplus by the definition of  $\bar{\Pi}$  while continuing to satisfy (IC) because  $\tilde{U}(\bar{\Pi}) > 0$ . A similar argument applies if  $\Pi_H < \bar{\Pi}$ . Hence,  $\Pi_L, \Pi_H \geq \bar{\Pi}$ . However, then  $\tilde{U}(\bar{\Pi}) + \bar{\Pi} = (p\bar{a} - c(\bar{a}))$ , where  $\bar{a}$  is the effort induced in any equilibrium with  $\Pi \geq \bar{\Pi}$  and  $U = \tilde{U}(\Pi)$ . Hence, any such payoffs can be sustained in a stationary equilibrium with effort  $\bar{a}$ . However, then  $\bar{a} < a^{FB}$ , since if  $\bar{a} = a^{FB}$ , (DE) requires (10) to hold. Consequently, (DE) binds for all  $\Pi \geq \bar{\Pi}$  and  $U = \tilde{U}(\Pi)$ . Since  $\tilde{U}(\Pi) > 0$ , there exists an interval of feasible  $\Pi > \bar{\Pi}$ . In particular, for the  $\Pi$  such that  $\tilde{U}(\Pi) = 0$ ,

$(1 - \delta)\frac{c(\bar{a})}{p} = \delta\Pi$ . We conclude that this result contradicts  $(1 - \delta)\frac{c(\bar{a})}{p} \geq \delta\bar{\Pi}$  and  $\bar{\Pi} < \Pi$ .

**Part 2: For any  $\Pi < \bar{\Pi}$ ,  $\Pi_L = \Pi < \Pi_H$ .** Denote the right-hand and left-hand derivatives of  $\tilde{U}$  by  $\partial_+\tilde{U}$  and  $\partial_-\tilde{U}$ , respectively, and note that  $\partial_-\tilde{U}(\Pi) \geq \partial_+\tilde{U}(\Pi)$  because  $\tilde{U}$  is concave.

We first claim that if  $\Pi < \bar{\Pi}$ , then  $\partial_+\tilde{U}(\Pi_L) \leq \partial_+\tilde{U}(\Pi) \leq \partial_-\tilde{U}(\Pi) \leq \partial_-\tilde{U}(\Pi_L)$ . Suppose  $\partial_+\tilde{U}(\Pi_L) > \partial_+\tilde{U}(\Pi)$ . Since  $\Pi < \bar{\Pi}$ ,  $\tilde{U}(\Pi) > 0$ . Thus, we can perturb the equilibrium by increasing  $\Pi_L$  without violating (IC). Doing so increases  $\Pi$  at rate  $(1 - p)\delta$  and decreases  $U$  at rate  $(1 - p)\delta\partial_+\tilde{U}(\Pi_L)$  and remains an equilibrium. Hence,  $\partial_+\tilde{U}(\Pi) \geq \frac{(1-p)\delta\partial_+\tilde{U}(\Pi_L)}{(1-p)\delta}$ , which contradicts our assumption. Similarly, if  $\partial_-\tilde{U}(\Pi_L) < \partial_-\tilde{U}(\Pi)$ , then we can prove contradiction by decreasing  $\Pi_L$ , which does not violate any constraint.

Next, we argue that  $\partial_+\tilde{U}(\cdot)$  is strictly decreasing for all  $\Pi < \bar{\Pi}$  which means that  $\partial_+\tilde{U}(\Pi_L) \leq \partial_+\tilde{U}(\Pi)$  and  $\partial_-\tilde{U}(\Pi) \leq \partial_-\tilde{U}(\Pi_L)$  together imply  $\Pi = \Pi_L$ . Towards contradiction, suppose that  $\tilde{U}(\cdot)$  is linear on an interval  $\Pi^A < \Pi^B$ . Let  $(b^i, a^i, \Pi_L^i, \Pi_H^i)$  be the bonus, action, and continuation payoffs associated with  $\Pi^i$ ,  $i \in \{A, B\}$ . Then, for any  $\alpha \in (0, 1)$ ,  $b = \alpha b^A + (1 - \alpha)b^B$ ,  $a = \alpha a^A + (1 - \alpha)a^B$ ,  $\Pi_L = \alpha\Pi_L^A + (1 - \alpha)\Pi_L^B$ , and  $\Pi_H = \alpha\Pi_H^A + (1 - \alpha)\Pi_H^B$  are an equilibrium. If  $a^A \neq a^B$ , then the worker's payoff from this convex combination is strictly larger than  $\alpha\tilde{U}(\Pi^A) + (1 - \alpha)\tilde{U}(\Pi^B)$  because  $c(\cdot)$  is strictly convex. Hence,  $a^A = a^B$ .

Now, (DE) must bind for any  $\Pi < \bar{\Pi}$ . Suppose it does not, so  $a^A = a^B \geq a^{FB}$ . Then, the upper bound of (LL) binds, since, otherwise, we could increase  $b$  in order to increase  $U$  while holding total surplus fixed, which would contradict  $\Pi < \bar{\Pi}$ . Moreover,  $\Pi_H \leq \bar{\Pi}$ , since otherwise we could decrease it without affecting total surplus, which similarly contradicts  $\Pi < \bar{\Pi}$ . Thus,

$$(1 - \delta)b = (1 - \delta)a \geq (1 - \delta)a^{FB} > \delta\bar{\Pi} \geq \delta\Pi_H,$$

where the equality follows from binding (LL), the first weak inequality follows from  $a \geq a^{FB}$ , the strict inequality follows by (10), and the second weak inequality follows because  $\Pi_H \leq \bar{\Pi}$ . We conclude that (DE) is violated, which proves a contradiction.

Since (DE) binds,

$$\Pi^A = (1 - \delta)y + \delta(1 - p)\Pi_L^A.$$

Furthermore,  $\Pi_L^A \geq \Pi^A$ , since if  $\Pi_L^A < \Pi^A$  then  $\partial_+ \tilde{U}(\Pi_L^A) > \partial_+ \tilde{U}(\Pi^A)$ , since  $\Pi^A$  is the left endpoint of the linear segment. By an analogous argument,  $\Pi_L^B \leq \Pi^B$ .

If  $\Pi_L^B < \Pi^B$ , we can increase  $\Pi_L^B$ , which contradicts that  $\Pi^B$  is the right endpoint of the linear segment. So  $\Pi_L^B = \Pi^B$ , and by a similar argument,  $\Pi_L^A = \Pi^A$ . But then

$$\Pi^A = \frac{1 - \delta}{1 - \delta(1 - p)} a^A = \frac{1 - \delta}{1 - \delta(1 - p)} a^B = \Pi^B,$$

contradicting  $\Pi^A < \Pi^B$ .

We have shown that  $\partial_+ \tilde{U}(\cdot)$  is strictly decreasing for all  $\Pi < \bar{\Pi}$ . Therefore,  $\Pi_L = \Pi$  on this range.

**Part 3: Effort, profit, and repayment dynamics.** We argue that in the equilibrium yielding payoffs  $(\tilde{U}(\Pi), \Pi) \in E$ ,  $a = a^*(\Pi)$  as defined in the statement of Proposition 3. Indeed, suppose (10) holds. Then  $a^*(\Pi) = a^{FB}$  for any  $\Pi \geq \bar{\Pi}$  and so  $a = a^*(\Pi)$  follows immediately. Otherwise, we argued in Part 2 that (DE) binds. Therefore,

$$\Pi = (1 - \delta)pa + \delta(1 - p)\Pi_L,$$

Since  $\Pi = \Pi_L < \bar{\Pi}$ ,  $a = \frac{1 - \delta(1 - p)}{(1 - \delta)p} \Pi < a^{FB}$  and so  $a = a^*(\Pi)$ .

For profit dynamics, we argue that  $\Pi_H > \Pi$  whenever  $\Pi < \bar{\Pi}$ . Suppose not; then  $\Pi_H < \bar{\Pi}$ . But then the upper bound of (LL) must bind. Otherwise, we could increase  $b$  and increase  $\Pi_H$  so that  $(1 - \delta)b + \delta\Pi_H$  is constant, which satisfies all constraints because  $\tilde{U}(\Pi_H) + \Pi_H$  is strictly increasing in this range. Therefore,

$$\Pi = \delta p \Pi_H + \delta(1 - p)\Pi_L \leq \delta \Pi,$$

since  $\Pi_H \leq \Pi$  by assumption and  $\Pi_L = \Pi$  by Part 2.

Finally, we argue that  $a_t$  converges to  $a_{\max}$  as  $t \rightarrow \infty$  with probability 1 in any profit-maximizing equilibrium. Since  $\Pi_L = \Pi$  and high output is realized with probability  $p > 0$  in each period, it suffices to show that with probability 1, any profit-maximizing equilibrium reaches a period in which  $\Pi_H \geq \bar{\Pi}$ . So long as  $\Pi_H < \bar{\Pi}$ , (LL) binds and so

$$\Pi_H - \Pi = \frac{1 - \delta}{\delta p} \Pi.$$

For any  $K > 0$ ,  $\Pi > 0$  in the first period of any equilibrium in which the project is funded. Therefore, the manager's profit increases by an amount that is bounded away from 0 every time high output is realized and remains constant following low output. So  $\Pi_H \geq \bar{\Pi}$  after a finite number of high outputs, which happen with probability 1 as  $t \rightarrow \infty$ . ■

## A.5 Proof of Corollary 1

**Fastest Repayment Path:** We first claim that this payment path repays the creditor as quickly as possible. Indeed, for any payment path such that  $b > 0$  in a period for which  $K(U_H, \Pi_H) > 0$ , we can decrease  $b$  and increase  $U_H$  so that  $(1 - \delta)b + \delta U_H$  remains constant, and increase  $r$  so that  $b + r$  remains constant. Lemma 1 implies that both worker and creditor earn the same expected surplus following this perturbation. However, perturbing the equilibrium in this way decreases  $K(U_H, \Pi_H)$  and therefore repays the creditor faster. Since both (DE) and the upper bound of (LL) bind whenever  $\Pi_H < \bar{\Pi}$  in any profit-maximizing equilibrium, the specified payment path is consistent with a profit-maximizing equilibrium.

**Slowest Repayment Path:** This payment path sets  $U = 0$  in each period, which clearly maximizes  $K(U, \Pi) = \tilde{U}(\Pi) - U$  in that period and so repays the creditor as slowly as possible. The proof of Proposition 3 shows that both (DE) and the upper bound of (LL) bind whenever  $\Pi_H < \bar{\Pi}$ . This payment path also satisfies the lower bound of (LL), since  $b = \frac{c(a)}{p} \geq 0$  and  $r = a - b \geq 0$  because  $pa - c(a) \geq 0$  for any  $a \leq a_{\max}$ . Therefore, these



payments are consistent with a profit-maximizing equilibrium. ■

## A.6 Proof of Proposition 4

Analogous to the set  $E$ , define

$$E^T \equiv \{(U, \Pi) | \exists K \geq 0 \text{ such that } (U, \Pi, K) \text{ are truth-telling equilibrium payoffs}\}.$$

Define the problem (P) as maximizing (5) subject to (PK-A)-(LL),  $(U_H, \Pi_H) \in E^T$  and  $(U_L, \Pi_L) \in E^T$ , and (TT), with the restriction to  $a \in \{0, y\}$ . Let  $K^T(U, \Pi)$  be the value function for (P).

This proof characterizes  $K^T(\cdot)$  with a series of lemmas, then uses that characterization to prove Proposition 4.

**Lemma 2** *Define*

$$E_1 \equiv \{(U, \Pi) \in E^T | \exists a \text{ solution to (P) with } a = y\} \subseteq E^T,$$

and let  $E_0 \equiv E^T \setminus E_1$ . Then  $(0, 0) \in E_0$  and  $K^T(0, 0) = 0$ , so that  $(0, 0)$  can be supported by liquidating the firm. Moreover, any  $(U, \Pi) \in E_0$  can be implemented by randomizing between  $(0, 0)$  and some  $(U', \Pi') \in E_1$ .

### Proof of Lemma 2

First, note that  $(0, 0) \in E_0$ , since otherwise (PK-P) would be violated. Consider any  $(U, \Pi)$  that can be implemented with  $a = 0$ . Then  $U_L = \frac{U}{\delta}$ ,  $\Pi_L = \frac{\Pi}{\delta}$ , and consequently  $K^T(U, \Pi) = \delta K^T(\frac{U}{\delta}, \frac{\Pi}{\delta})$ . In particular,  $K^T(0, 0) = \delta K^T(0, 0)$  and so  $K^T(0, 0) = 0$ , which can be attained through liquidation.

Consider  $(U, \Pi) \in E_0$  with  $(U, \Pi) \neq (0, 0)$ . If  $(U, \Pi)$  can be implemented with  $a = 0$ , then  $K^T(U, \Pi) = \delta K^T(\frac{U}{\delta}, \frac{\Pi}{\delta}) = \delta K^T(\frac{U}{\delta}, \frac{\Pi}{\delta}) + (1 - \delta)K^T(0, 0) \leq K^T(U, \Pi)$ , where the first

equality follows by the argument above, the second follows because  $K^T(0,0) = 0$ , and the inequality holds because  $K^T$  is concave. So  $K^T(U, \Pi)$  is linear between  $(0,0)$  and  $(\frac{U}{\delta}, \frac{\Pi}{\delta})$  for any  $(U, \Pi) \in E_0$ . Let  $(U', \Pi')$  be the right endpoint of this linear segment, and note that  $(U', \Pi') \in E_1$ . Therefore, any  $(U, \Pi) \in E_0$  can be implemented by randomizing between liquidation and some  $(U', \Pi') \in E_1$ .

Finally, if  $(U, \Pi) \in E_0$  can be implemented with  $a$  such that  $0 < \Pr\{a = y\} < 1$ , then  $(U, \Pi)$  can also be implemented by randomizing between a point in  $E_1$  and a point that can be implemented with  $a = 0$ . So by the above argument, such  $(U, \Pi)$  can also be implemented by randomizing between continuation and liquidation. ■

Now, define

$$\begin{aligned}\Pi_{\max} &= py - c; \\ \Pi_f &= \frac{p(1-\delta)y}{1-(1-p)\delta}; \\ \tilde{U}^T(\Pi) &= \max \{U | (\Pi, U) \in E^T\}.\end{aligned}$$

Note that (8) implies that  $\Pi_f \leq \Pi_{\max}$ .

**Lemma 3** For any  $(U, \Pi) \in E^T$ ,

1.  $U + \Pi = \Pi_{\max}$  if and only if  $\Pi \in [\Pi_f, \Pi_{\max}]$  and  $U = \tilde{U}^T(\Pi)$ ;
2. If  $U + \Pi < \Pi_{\max}$ , then  $K^T(U, \Pi) + U + \Pi < py - c$ .

### Proof of Lemma 3

Suppose that  $U + \Pi = \Pi_{\max}$ . Then  $\Pi_{\max}$  is the maximum feasible total surplus, so  $K^T(U, \Pi) = 0$  and hence  $U = \tilde{U}^T(\Pi)$ . Define  $\Pi'_f$  as the manager's smallest equilibrium payoff such that  $\tilde{U}^T(\Pi'_f) + \Pi'_f = \Pi_{\max}$ , and denote  $\Pi_H$  and  $\Pi_L$  as the associated continuation profits. Note that  $a = y$  for  $(\tilde{U}^T(\Pi'_f), \Pi'_f)$ , and so

$$\begin{aligned}\Pi'_f &= p((1-\delta)(y-b) + \delta\Pi_H) + (1-p)\delta\Pi_L \\ &\geq p(1-\delta)y + (1-p)\delta\Pi'_f,\end{aligned}$$

where the equality holds by (PK-A) and the inequality follows because (DE) implies  $\delta\Pi_H \geq (1 - \delta)b$ , and  $\Pi_L \geq \Pi'_f$  in order for sum of the manager's and worker's payoffs to equal  $\Pi_{\max}$ . Rearranging this expression yields  $\Pi'_f \geq \Pi_f$ .

Now, suppose  $\Pi \geq \Pi_f$ , and consider the set of stationary strategies such that  $a = 1$ ,  $r = 0$  and  $b \in \left[\frac{c}{p}, \frac{\delta\Pi_f}{1-\delta}\right]$ . It is straightforward to argue that all of these payments can be sustained in a relational contract. With  $b = \frac{c}{p}$ , the manager earns  $\Pi_{\max}$ ; with  $b = \frac{\delta\Pi_f}{1-\delta}$ , the manager's payoff is  $\Pi_f$ . Therefore,  $\tilde{U}^T(\Pi) + \Pi = py - c$  for any  $\Pi \geq \Pi_f$ . Combined with the result that  $\Pi'_f \geq \Pi_f$ , we conclude that  $\Pi'_f = \Pi_f$  and that for any  $\Pi \in [\Pi_f, \Pi_{\max}]$ ,  $\tilde{U}^T(\Pi) + \Pi = py - c$ , which proves part 1.

Next, define

$$z \equiv \min \{U + \Pi | K^T(U, \Pi) + U + \Pi = \Pi_{\max}\}.$$

Suppose  $z < \Pi_{\max}$ , and choose  $(U, \Pi) \in E^T$  such that  $U + \Pi = z$  and  $K^T(U, \Pi) + U + \Pi = py - c$ . Then it must be that  $a = 1$  with probability 1, and moreover  $U_L + \Pi_L + K^T(U_L, \Pi_L) = py - c$ . Then summing (PK-A) and (PK-P) implies that

$$\begin{aligned} z &= (1 - \delta)(py - c - pr) + \delta p(U_H + \Pi_H - U_L - \Pi_L) + \delta(U_L + \Pi_L) \\ &\geq (1 - \delta)(py - c) + \delta(U_L + \Pi_L), \end{aligned}$$

where the inequality follows from (TT). Since  $z < py - c$ ,  $U_L + \Pi_L < z$ . But  $U_L + \Pi_L + K^T(U_L + \Pi_L) = py - c$ , yielding a contradiction. ■

**Lemma 4** *The following hold:*

1. For  $\Pi \leq \Pi_f$ ,

$$\tilde{U}^T(\Pi) = \frac{\tilde{U}^T(\Pi_f)}{\Pi_f} \Pi.$$

2. For any  $\Pi \in [0, \Pi_{\max}]$ ,  $K^T(\tilde{U}^T(\Pi), \Pi) = 0$ .

3. For any  $(U, \Pi)$  with  $U + \Pi < py - c$ ,  $K^T(U, \Pi) + \Pi$  is strictly increasing in  $\Pi$ . For  $\Pi \geq \Pi_f$ ,  $K^T(U, \Pi) + U$  is strictly increasing in  $U$ .

**Proof of Lemma 4**

**Part 1:** Define  $\xi = \max \left\{ \frac{U}{\Pi} \mid (U, \Pi) \in E^T \right\}$  and let  $(U, \Pi)$  be such that  $\frac{U}{\Pi} = \xi$ . Lemma 2 implies that we can take  $(U, \Pi) \in E_1$ , and it is immediate from the definition of  $\tilde{U}^T(\cdot)$  that we can take  $U = \tilde{U}^T(\Pi)$ . We claim that  $\Pi = \Pi_f$ . For any  $\Pi > \Pi_f$ ,  $\tilde{U}^T(\Pi) + \Pi = \Pi_{\max}$ , and so  $\tilde{U}^T(\Pi)$  is strictly decreasing in  $\Pi$  on this range. So  $\frac{\tilde{U}^T(\Pi_f)}{\Pi_f} > \frac{\tilde{U}^T(\Pi)}{\Pi}$  for any  $\Pi > \Pi_f$ .

Consider  $\Pi < \Pi_f$ . To show this, we note two properties. First,  $r = 0$  in any equilibrium with payoffs  $(\tilde{U}^T(\Pi), \Pi)$ , since otherwise we could decrease  $r$  and increase  $b$  so that  $r + b$  is constant and the worker earns a strictly higher payoff than  $\tilde{U}^T(\Pi)$ .

Second, we claim that there exists an equilibrium giving  $(\tilde{U}^T(\Pi), \Pi)$  in which  $\Pi_H > \Pi_f$ . Recall that  $(\tilde{U}^T(\Pi), \Pi) \in E_1$ , and note that either (DE) or the upper bound of (LL) must bind, since otherwise we could increase  $b$  and hence increase the worker's payoff. Suppose the upper bound of (LL) is slack, so  $b < y$ . From the public randomization device,  $\tilde{U}^T(\Pi)$  is concave, so  $\Pi + \tilde{U}^T(\Pi)$  is increasing in  $\Pi$  (because it is constant for  $\Pi \geq \Pi_f$ ). Therefore, consider increasing  $b$  and  $\frac{\delta}{1-\delta}\Pi_H$  by the same amount. Doing so holds the manager's payoff constant and gives the worker a higher payoff. So there exists an equilibrium giving  $(\tilde{U}^T(\Pi), \Pi)$  in which  $b = y$ . But then

$$\delta\Pi_H \geq (1 - \delta)y > \frac{\delta p(1 - \delta)y}{1 - (1 - p)\delta} = \delta\Pi_f,$$

as desired.

Given these two properties,

$$\begin{aligned}
1 + \xi &= \frac{U + \Pi}{\Pi} \\
&= \frac{(1-\delta)(py-c) + \delta p(py-c) + \delta(1-p)(U_L + \Pi_L)}{\Pi} \\
&\leq \frac{(1-\delta + \delta p)(py-c) + (1-p)\Pi_L(1+\xi)}{\Pi} \\
&\leq \frac{(1-\delta + \delta p)(py-c) + (1-p)\Pi_L(1+\xi)}{(1-\delta)py + \delta(1-p)\Pi_L} \\
&\leq \frac{(1-\delta + \delta p)(py-c)}{(1-\delta)py} \\
&= \frac{\tilde{U}^T(\Pi_f) + \Pi_f}{\Pi_f}
\end{aligned} \tag{11}$$

Here, the first equality follows from  $\Pi_H > \Pi_f$  and so  $\tilde{U}^T(\Pi_H) + \Pi_H = py - c$ , the first inequality holds because  $\frac{U_L + \Pi_L}{\Pi_L} \leq 1 + \xi$  by definition of  $\xi$ , the second inequality follows because (DE) implies that  $\Pi \geq (1 - \delta)py + \delta(1 - p)\Pi_L$ , the third inequality holds because  $\delta(1 - p)\Pi_L \geq 0$ , and the final equality holds by definition of  $\Pi_f$  and because  $\tilde{U}^T(\Pi_f) + \Pi_f = py - c$ .

We conclude that  $\frac{\tilde{U}^T(\Pi_f)}{\Pi_f} = \xi$ , as desired, which implies part 1 of Lemma 4 because  $\tilde{U}^T(\Pi)$  is concave and so  $\frac{\tilde{U}^T(\Pi)}{\Pi}$  is decreasing in  $\Pi$ , and strictly so unless  $\tilde{U}^T(\cdot)$  is linear.

**Part 2:** Note that for  $\Pi \geq \Pi_f$ , Lemma 3 implies that  $K^T(\tilde{U}^T(\Pi), \Pi) = 0$ . For  $\Pi < \Pi_f$ , (11) holds with equality only if  $\frac{U_L}{\Pi_L} = \xi$ . But then  $\Pi_L \geq \Pi_f$ , implying that  $K^T(U_L, \Pi_L) = 0$ , and similarly  $\Pi_H \geq \Pi_f$  so  $K^T(U_H, \Pi_H) = 0$ . Since  $r = 0$  as well,  $K^T(\tilde{U}^T(\Pi), \Pi) = 0$  in this range too.

**Part 3:** Lemma 3 and the concavity of  $K^T(\cdot)$  imply that  $K^T(U, \Pi) + U$  is strictly increasing in  $U$  for  $\Pi \geq \Pi_f$ . Similarly, Lemma 3, concavity of  $K^T$ , and the fact that  $\tilde{U}^T(\Pi)$  is maximized at  $\Pi_f$  imply that  $K^T(U, \Pi) + \Pi$  is strictly increasing whenever  $U + \Pi < py - c$ . ■

Given this characterization, we are prepared to prove our main result.

**Proof of Proposition 4**

**Part 1:** Suppose that  $\Pi \geq \frac{p(1-\delta)y}{1-(1-p)\delta}$  and  $K^T(U, \Pi) > 0$ . Since  $U + \Pi + K^T(U, \Pi) \leq \Pi_{max} \equiv py - c$ ,  $U + \Pi < \Pi_{max} \equiv py - c$ . Part 2 of Lemma 3 therefore implies that

$$K^T(U, \Pi) + U + \Pi < py - c.$$

Now, part 1 of Lemma 3 implies that  $K^T(\tilde{U}^T(\Pi), \Pi) + \tilde{U}^T(\Pi) + \Pi = py - c$ , where  $\tilde{U}^T(\Pi) > U$  by definition. Since

$$K^T(\tilde{U}^T(\Pi), \Pi) + \tilde{U}^T(\Pi) + \Pi > K^T(U, \Pi) + U + \Pi$$

and  $K^T(\cdot)$  is weakly concave, we conclude that

$$K^T(U', \Pi) + U' + \Pi > K^T(U, \Pi) + U + \Pi$$

for any  $U' > U$ .

**Part 2:** Suppose  $U_H + \Pi_H < py - c$ . First, we argue that  $r + b = y$  whenever  $U_H + \Pi_H < py - c$ . Note that  $\Pi_H < \Pi_f$ . Suppose  $r + b < y$ , and consider an alternative that increases  $r$  by  $\epsilon > 0$  and increases  $\Pi_H$  by  $\frac{1-\delta}{\delta}\epsilon$ . For  $\epsilon > 0$  sufficiently small, this perturbation is feasible—in particular, Lemma 4 implies  $(U_H, \Pi_H + \frac{1-\delta}{\delta}\epsilon) \in E^T$  because  $\Pi_H < \Pi_f$ —and it continues to satisfy the constraints of (P). Moreover,

$$\delta K^T\left(U_H, \Pi_H + \frac{1-\delta}{\delta}\epsilon\right) + (1-\delta)\epsilon > \delta K^T(U_H, \Pi_H)$$

by part 3 of Lemma 4. So the original equilibrium cannot be on the frontier defined by  $K^T(\cdot)$ ; contradiction.

Next, we show that  $b = 0$ . Suppose  $b > 0$ , and consider increasing  $r$  and decreasing  $b$  by  $\epsilon > 0$ , and increasing  $U_H$  by  $\frac{1-\delta}{\delta}\epsilon$ . As before, this perturbation satisfies the constraints of

(P). It is also feasible for sufficiently small  $\epsilon > 0$ , since  $\Pi_H > \Pi_f$  from the proof of Lemma 4 and so  $U_H < py - c - \Pi_H \leq \tilde{U}^T(\Pi_H)$ .

Now, since  $\Pi_H > \Pi_f$ ,

$$\delta\epsilon + \delta K\left(U_H + \frac{1-\delta}{\delta}\epsilon, \Pi_H\right) > \delta K(U_H, \Pi_H)$$

by part 3 of Lemma 4. So the creditor earns a strictly higher payoff in the perturbed equilibrium. Hence,  $b = 0$  and  $r = y$ .

**Part 3:** By Lemma 2, it suffices to show that when  $K(U, \Pi) > 0$ , continuation play at some successor history lies in  $E_0$  with positive probability. Suppose not; then  $K(U, \Pi) + U + \Pi = py - c$ . But then Lemma 3 (part 1) and Lemma 4 (part 2) imply that  $K(U, \Pi) = 0$ ; contradiction. ■

## B Data Appendix

### B.1 German Administrative Data

We use two datasets from the Federal Data Center (FDZ) of the Institute for Employment Research (IAB) of German Employment Agency: the Linked Personnel Panel (LPP) survey of firms and workers and the IAB Establishment Panel (IAB-EP), a survey of firms. To access the desired waves of both datasets, we combine two FDZ products: LPP-ADIAB Version 1975–2014 and LIAB. LPP-ADIAB is the LPP survey data that is linked to the administrative data of the IAB. The LIAB is the Longitudinal Model (version 1993–2014) of the Linked Employer-Employee Data from the IAB. The data were accessed on-site at the Research Data Centre of the Federal Employment Agency at the Institute for Employment Research (FDZ) and via remote data access at the FDZ. The documentation for LPP-ADIAB is available in Broszeit et al. (2017), and the documentation for LIAB is available in Heining et al. (2016). The webpage of the Federal Data Center (<https://fdz.iab.de/en/>) contains the questionnaires and further details about both datasets.

The LPP data consists of two waves. In both of them, around 6,000 workers were interviewed, 3,000 of whom were interviewed in both waves. These workers are selected from approximately 1,000 establishments from the IAB-EP sample. We obtain information about leverage from the 2008 wave of the IAB-EP, which does not contain all of the relevant firms. After linking surveyed individuals to our data on leverage, we have around 7,600 observations. We do not otherwise restrict the sample, although the number of observations varies slightly across regressions because some of the selected variables are missing for some observations.

Using the LPP data, we define 3 outcome variables that we use in Table 2. First, “considers quitting” is based on question “How many times in the past 12 months have you thought about changing your job?”. The raw answers are (1) daily, (2) a few times a week, (3) a few times a month, (4) a few times a year, (5) never. We recode the variable



by subtracting the value from 6 so that higher numbers correspond to a higher intensity of considering quitting. Second, “commitment” is based on 6 statements from the “commitment” section of the survey. For each question, the respondent employee chooses one of 5 answers: (1) fully applies; (2) largely applies; (3) neutral; (4) does rather not apply; and (5) does not apply at all. An example statement is, “I would be very happy to spend the rest of my career with this organization.” For each statement, we again recode the answers so that higher values correspond to higher commitment. Finally, “job satisfaction” is based on the question “How satisfied are you today with your job? Please answer on a scale from 0 to 10, where 0 means ‘totally unhappy’ and 10 means ‘totally happy.’” The main independent variable in the worker-level regressions, as well as some of the control variables, come from IAB-EP, which we will discuss further in the next paragraph. Additional controls include the workers’ sex and year of birth, a categorial measure of their educational level, their income level (five quintiles), and an indicator for their contract with the firm having a fixed term (instead of a permanent contract).

The IAB-EP is an annual survey that randomly samples around 16,000 establishments. Firms are surveyed repeatedly, but each year, some firms stop participating and other firms are added. The survey includes modules on firm personnel, investment, business practices, and other topics. Some questions are repeated every year, while others are only asked in selected years. For our regressions, questions about staffing problems are asked every 2 years, while the question about firm leverage was asked only in 2005 and 2008.

Our measure of leverage is based on the question, “Which of the following sources of finance were used for the (reported) total investment? Please indicate how this amount is distributed across the different sources: (...) Private credit (from banks, credit unions, saving banks, enterprises) (...) Approx. .... %”. That is, this measure gives the share of investment in a given year financed with debt, as opposed to current receipts, other equity, and subsidies. Measures of staffing problems are based on the question: “What kind of problems with human resources management do you expect for your establishment/office during the next

two years? Please tick where applicable in the list!", with possible answers including "lack of motivation in the workplace" and "high turnover." Our additional controls from this survey include average wage in the establishment (total wagebill divided by number of employees), relative employment growth (change in employment divided by total employment), type of management (by owner, by professional manager, mixed) and profitability, which is measured according to the three-category scale "positive," "approximately balanced," or "negative." We perform robustness checks with an alternative five-category profitability scale "very good", "good", "satisfactory", "sufficient", or "unsatisfactory."

## **B.2 Amadeus Data**

Our regressions on wages and productivity use the Amadeus dataset, which we access using WRDS. Since the required variables are often unavailable for small firms, we include only large and very large companies. A firm is considered large or very large if it meets at least one of the following conditions: more than 150 employees, operating revenue higher than 10 mln EUR, or total assets of 20 mln EUR or more. We use the entire available time period, 1985–2017, but since Amadeus provides no more than 10 years of data for each firm, for most of firms our data comes from between the 2006/2007 and the 2016/17 waves. We keep the consolidated data whenever both the consolidated and unconsolidated data are available. If a firm has more than one observation per year, we use the latest.

Capital is defined as the log of fixed assets, and labor is defined as the log of employee costs. We use operating revenue as a proxy for output since sales entails more missing data. Our proxy for intermediate inputs is the log of material costs, and the industry is given by 2-digits codes from the NACE2 classification. We keep only manufacturing firms, which have NACE2 codes between 10 and 32, and we drop firms that have fewer than 5 yearly observations of capital. To reliably estimate TFP by (industry X country), we keep only those industry-country pairs that have at least 1000 firm-year observations with non-missing capital, labor, output, and intermediate input measures. Our results with OLS-based TFP-R

are similar if we instead use a threshold of 100 observations; our other measures, however, are more data-intensive, so they frequently fail to converge if the number of observations is too small.

This restriction greatly reduces our working sample for two reasons. First, not all information is available for all countries. For example, firms in the United Kingdom do not report intermediate outputs, so they are dropped. Second, smaller countries or industries might not have 1000 observations. Nevertheless, all but one industry (tobacco manufacturing) is included for at least one country, and all large European countries, except for the United Kingdom, are included in the final sample. Our final data has 127,703 observations. To ensure that our results are not driven by outliers, we trim both regressands and regressors at the 5th and 95th percentile. Table 4 presents summary statistics for our final sample.

Table 4: Summary Statistics  
 PANEL A: SUMMARY STATISTICS FOR GERMAN ADMINISTRATIVE DATA:

Firm-Level Data	Mean	Median	St Dev	Min	Max
Leverage	15.4	0	30.4	0	100
Leverage>0	0.250	0	0.433	0	1
Problems with Motivation	0.072	0	0.259	0	1
Problems with Turnover	0.029	0	0.168	0	1
Num of Employees	164.4	21	856.2	0	49723
Employment Growth	-0.094	0	13.916	-3449	1
Profitability	1530	1454.5	1166.2	0	15000
Average Wage	4.03	5	1.63	1	5
Obs Cross-Section (Lev):	14 625				
Obs Panel (Lev):	19 227				
Worker-Level Data					
Considers Quitting	1.59	1	0.92	1	5
Commitment to Firm	3.72	3.83	0.89	1	5
Job Satisfaction	7.54	8	1.71	1	5
Female	0.28	0	0.45	0	1
Year of Birth	1966.6	1965	10.3	1945	1996
Education Level	3.29	3	1.13	1	8
Income Level	2.97	3	1.41	1	5
Fixed-Term Contract	0.050	0	0.218	0	1
Observations	11 809				

PANEL B: SUMMARY STATISTICS FOR AMADEUS DATA

	Mean	Median	Std Dev	Min	Max
Leverage	0.140	0.114	0.112	0	0.553
$\Delta$ Leverage	-0.0024	-0.0024	0.0416	-0.1314	0.1472
Operating revenue (mln Euro)	60.0	25.3	125.7	0.5	8277.8
Fixed assets (Mln Euro)	16.1	6.4	27.5	0.3	223.5
Costs of employees (Mln Euro)	9.7	4.1	26.6	0.0	4115.3
Number of employees	140.18	90	135.36	15	775
Material costs (Mln Euro)	33.3	12.7	76.5	0.0	2733.5
Total assets (Mln Euro)	45.7	21.7	78.2	0.4	2756.6
TFP-R (GNR)	23.348	23.343	0.676	15.824	26.043
$\Delta$ TFP-R (GNR)	0.021	0.017	0.189	-5.363	5.524
Observations	127703				

Panel A presents summary statistics from LPP and IAB-EP data. Statistics for firm-level variables refer to the full dataset which contains values from 2004-2008 period. For panel regressions, only 2 observations per firm are used (2005/2006 and 2008). For cross-sectional regressions, one observatin per firm - averaging values from the whole period - is used. For worker-level data, summary statistics come from the pooled cross-section of workers from two waves of LPP survey. For firm-level data, number of observations refers to number of non-missing leverage observations; for worker data, it refers to all observations among those that are merged with firm-level leverage measureare used in regressions. Panel B presents summary statistics from Amadeus data are presented. The sample contains all firms with more than 150 employees, operating revenue higher than 10 million EUR or total assets of 20 million EUR or more. Observations come from 1985-2017, but each firm has at most 10 observations, usually from 2006/07-2016/17.

## C Internet Appendix: Collusion and Truth-telling

This section defines an equilibrium refinement that captures collusion between the manager and worker, then shows that Definition 1 is sufficient to incorporate this notion of collusion. This sufficiency argument relies on binary effort.

**Definition 2** *A SPE  $\sigma^*$  is a collusion equilibrium if, after the creditor accepts the equilibrium formal contract  $(R, l(\cdot))$ , continuation play maximizes the manager's payoff among all continuation SPE.*

The intuition for a collusion equilibrium follows from the following heuristic timing. Suppose that the manager first negotiates with the creditor to secure a loan. After the creditor signs this formal contract, however, the manager can sit down with the worker and propose a continuation equilibrium. During her negotiations with the creditor, the manager cannot credibly promise to choose an equilibrium in which the worker punishes her for failing to repay the creditor. Therefore, Definition 2 captures the idea that the creditor does not have a seat at the table when the manager and worker decide on a relational contract. Essentially, such equilibria resemble “multi-tier” contracting problems (Tirole (1986); DeMarzo et al. (2005)), with the important differences that the game is infinite-horizon and contracts must be self-enforcing.

One immediate implication of this definition is that liquidation must occur with positive probability whenever the project is funded in a collusion equilibrium. If it did not, then the manager and worker could agree to never repay the creditor for her initial loan. We prove a stronger result: there exists a profit-maximizing truth-telling equilibrium that is also a collusion equilibrium.

**Proposition 5** *Consider the model with binary effort from Section 5. There exists a profit-maximizing truth-telling equilibrium that is a collusion equilibrium.*

**Proof:** Proposition 4 says that there exists a truth-telling equilibrium in which  $a_t = y$  in every period until the project is liquidated. Consider this equilibrium and note that it maximizes *total* surplus given the formal contract  $l(\cdot)$ . It suffices to show that, immediately after the creditor agrees to  $l(\cdot)$ , no alternative equilibrium gives the manager a strictly higher expected continuation payoff.

Consider the following “ancillary game,” which has two players: a firm and a creditor. In each period, the firm chooses  $a_t$ ,  $r_t$ , and  $b_t$ , bears the cost  $ca_t$ , and earns the sum of the manager and worker’s utility. The creditor’s actions and payoffs are unchanged. Fix  $l(\cdot)$  as in the original game. Since the firm earns  $\Pi + U$  in each period, (7) implies that it has no one-shot deviation in  $r_t$ . It has no one-shot deviation in  $a_t$  either since, regardless of  $l(\cdot)$ , the actions  $a_t = 1$  and  $r_t = 0$  generate a strictly higher sum of manager and worker utilities than  $a_t = r_t = 0$ . Finally, it has no deviation in  $b_t$ , which does not affect the sum of the manager’s and worker’s payoff. The one-shot deviation principle applies to this ancillary game, so the firm’s payoff is maximized by choosing  $a_t$ ,  $r_t$ , and  $b_t$  as specified in the collusive-proof equilibrium.

Now, return to the three-player game. The preceding argument implies that after the creditor agrees to  $l(\cdot)$ ,  $\Pi + U$  is maximized by following the equilibrium strategy. However,  $U = 0$  immediately after the creditor agrees to  $l(\cdot)$ . Since  $U \geq 0$ ,  $\Pi$  is bounded above by  $\Pi + U$ , and moreover, we have argued that  $\Pi = \Pi + U$  at the point where  $\Pi + U$  is maximized. Therefore,  $\Pi$  is maximized by following the equilibrium, so there exists no alternative equilibrium that generates strictly a higher profit given  $l(\cdot)$ . The manager is willing to choose  $l(\cdot)$  in equilibrium if all other contracts lead to the worker choosing  $a_t = 0$  in every  $t$ . We conclude that this profit-maximizing truth-telling equilibrium is a collusion equilibrium, as desired. ■

Proposition 5 exhibits a profit-maximizing truth-telling equilibrium that also satisfies Definition 2. However, this result does not say that this equilibrium is a *profit-maximizing* collusion equilibrium. It is in this sense that Definition 1 is a sufficient but *not* a necessary

condition for a collusion equilibrium—the manager can certainly do at least as well in a collusion equilibrium, but it is an open question whether she could do strictly better.