

Productivity and Debt in Relational Contracts

Daniel Barron, Jin Li, and Michał Zator*

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Abstract

We study how financial obligations constrain incentives and generate wage, debt, and productivity dynamics. A manager with limited liability simultaneously repays a creditor and motivates a worker. If the manager cannot commit to output-contingent payments, then debt constrains the worker's equilibrium incentives and both effort and pay increase as that debt is repaid. The manager might defer worker pay in order to repay its debts as quickly as possible, in which case debt has a delayed effect on productivity. If worker and manager can collude, then the creditor must threaten liquidation to induce repayment, leading to inefficient liquidation on the equilibrium path. Empirically, we show that increases in leverage are correlated with decreases in wages, current productivity, and future productivity. JEL Codes: C73, D21, D86, G32.

Keywords: Relational Contract, Productivity, Debt

*Barron (corresponding author): Northwestern University Kellogg School of Management, Global Hub, 2211 Campus Drive, Evanston IL 60208; email: d-barron@kellogg.northwestern.edu. Li: University of Hong Kong, 936 KK Leung Building, the University of Hong Kong; email: jli1@hku.hk. Zator: Northwestern University Kellogg School of Management, Global Hub, 2211 Campus Drive, Evanston IL 60208; email: michal.zator@kellogg.northwestern.edu. We are very grateful to Nicola Bianchi, Vicente Cuñat, Sebastian DiTella, Matthias Fahn, Michael Fishman, Willie Fuchs, Bob Gibbons, Hugo Hopenhayn, Niko Matouschek, David Matsa, Michael Powell, Michael Raith, and Andy Skrzypacz for tremendously helpful comments, as well as audiences at FOM, SIOE, and USC Marshall. We thank Amit Gandhi, Salvador Navarro, and David Rivers for sharing the Stata code for their productivity estimation. This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors. None of the authors have any declarations of interest. Any errors are ours.

1 Introduction

Managers borrow money in order to pursue new opportunities and grow their businesses, but even an attractive-looking investment is likely to fail unless employees work hard implementing it. A successful firm must therefore motivate its workers even as it repays its debts. To do so, a firm must overcome a common contractual friction that plagues both of these relationships: contractible measures of performance are not reliable because the firm can typically divert revenue into privately beneficial but unprofitable projects.¹ Payments to both workers and creditors must therefore be credible in the context of those ongoing relationships.²

In this paper, we explore how a manager’s financial obligations constrain her relationships with employees and thereby shape wage, debt, and productivity dynamics. We show that the manager’s financial obligations limit the incentives she can promise her workers, since an indebted manager is less willing to follow through on promised incentives. Consequently, effort declines when a firm takes on debt and slowly recovers as that debt is repaid. In a profit-maximizing relational contract, debt both depresses contemporaneous effort and potentially leads the manager to defer worker compensation, in which case debt continues to (temporarily) depress effort even after it has been fully repaid. We complement this analysis by documenting several stylized facts about debt, wages, and productivity that are consistent with our theory.

As an example of how borrowing constrains relational incentives, consider Lincoln Electric’s decision to significantly increase its leverage in the early 1990s. At the time, Lincoln Electric was struggling to recover from a devastatingly unprofitable international expansion. The company paid smaller discretionary bonuses to its U.S. workers in order to prioritize repaying its mounting debts, a decision that threatened to undermine its strong relational incentive system. In a radical departure from the company’s cooperative worker-manager relations, Lincoln workers openly voiced their “disgruntlement” about small bonuses, while managers expressed fears that the entire incentive system might “unravel” if they did not pay

¹Shleifer and Vishny (1997) discusses theory and evidence for these kinds of commitment problems in credit relationships, while Malcomson (2013) discusses similar commitment problems in agency relationships.

²See Bull (1987) and Levin (2003) on commitment problems in agency relationships and Aghion and Bolton (1992), Hart (1995), and Holmstrom and Tirole (1997) for commitment problems in credit relationships.

bonuses (Feder (1994); Hastings (1999)). Consistent with this example, Bae et al. (2011) shows that firms that treat their employees fairly tend to maintain low debt ratios; Strelbulaev and Yang (2013) documents that many U.S. firms have no or very little debt and that these firms perform well by a variety of measures; and Fahn et al. (2017) argues that firms which rely on relational contracts tend to have lower leverage.

In our model, a liquidity-constrained manager borrows money from a creditor to fund a project and then repeatedly motivates a worker to exert profit-increasing effort. The manager uses realized profits to repay the creditor and compensate the worker. The manager cannot commit to repay the creditor or reward the worker—for example, because she can divert cash for her private benefit—so promised payments are made credible by the threat that the worker might shirk or the creditor might liquidate the project if they are not paid.

The manager is willing to pay both her creditor and her worker because they will otherwise deny her future profits, which means that she will renege on *both* parties if she finds her *aggregate* promises too onerous. An indebted manager must promise large payments to the creditor, which limits the extent to which she can credibly reward the worker’s effort. Debt therefore depresses output. In a profit-maximizing relational contract, the manager’s outstanding debt decreases over time, with concomitant increases in effort. The profit-maximizing relational contract consequently ties debt dynamics to both compensation and productivity dynamics.

Section 3 shows how borrowing generates effort, wage, and debt dynamics in profit-maximizing equilibria. We show that the worker and the creditor essentially act as a single player in these equilibria, in the sense that current productivity depends on the *sum* of their continuation payoffs. The worker initially exerts little effort after the firm takes on debt; effort stochastically increases as the manager repays the creditor until it converges to a steady state that is independent of the initial borrowing. All payment schemes that support these productivity dynamics share the feature that the worker’s compensation is at least weakly backloaded. If this backloading is strict, then effort is negatively related to both *current* and *past* debt, with the implication that borrowing might depress firm productivity even after it has been repaid.

A key feature of profit-maximizing equilibria is that the worker and creditor both punish the manager “on one another’s behalf,” in the sense that both punish the manager if she

reneges on either of them.³ While it is profit-maximizing for the worker and creditor to coordinate their punishments in this way, practical obstacles might prevent them from doing so. In particular, Section 4 shows that the manager and worker have an incentive to collude in order to undermine coordinated punishments. In contrast to Section 3, the worker does not punish the manager “on the creditor’s behalf” in an equilibrium with collusion, so the creditor must rely on liquidation threats to induce repayment. But liquidation cannot condition on realized output, so it can occur on the equilibrium path while the manager is still repaying the creditor. To minimize the probability of liquidation, the manager optimally pays the creditor first and defers worker compensation, which strengthens Section 3’s claim that debt can have both immediate and lingering effects on compensation.

Section 5 studies the empirical relationship between debt, wages, and productivity. Using data from Europe, we show that increases in a firm’s book leverage are correlated with decreases in that firm’s wages, contemporaneous productivity, and productivity in the following year. These correlations hold after including either firm and year or industry-year fixed effects. We also find suggestive evidence that these relationships are strongest in industries where employees contribute the most to production and in countries where relational contracts are particularly valuable. Though by no means conclusive, these correlations are consistent with our mechanism’s predictions about how debt affects labor practices.

Our model builds on the relational contracting literature (Bull (1987); MacLeod and Malcolmson (1989); Levin (2003)) but applies these ideas to a manager who simultaneously manages relationships with both workers and creditors. Levin (2002) studies relational contracts with multiple agents but assumes transferable utility so that stationary equilibria are optimal. Recently, papers have considered dynamics that arise from liquidity constraints (Fong and Li (2017)), private information (Halac (2012); Malcolmson (2016)), or subjective evaluation (Fuchs (2007)). Among these papers, Board (2011) studies multilateral contracts in the presence of liquidity constraint but does not consider financing relationships. Andrews and Barron (2016) and Barron and Powell (2018), which study relational contracts with uncoordinated punishments, are related to our Section 4, but dynamics in those papers are driven by private monitoring rather than the possibility of collusion.

³Such punishments can also be interpreted as “whistleblowing:” the worker communicates any deviation to the creditor, who punishes by liquidating the firm. Regardless of interpretation, the key feature of these equilibria is that, because the worker observes all actions, he can target his punishments so that they occur only after the principal has truly deviated.

A vast literature studies contracting frictions in financial relationships (Jensen and Meckling (1976); Myers (1977); Townsend (1979); Gale and Hellwig (1985); Stein (2003)). In particular, Aghion and Bolton (1992), Hart (1995), Holmstrom and Tirole (1997), and others focus on commitment problems in finance contracts. We build on this approach to argue that the same commitment problems that plague financial relationships simultaneously constrain within-firm incentives. More recently, He and Milbradt (2016), DeMarzo and He (2017), and the papers discussed in the next paragraph consider dynamic models of financing with limited commitment. Our analysis of collusion relates to Tirole (1986), Biais and Gollier (1997), and other static models of collusion.

Our analysis is particularly related to Thomas and Worrall (1994), which studies dynamics in foreign investment, and Albuquerque and Hopenhayn (2004), Clementi and Hopenhayn (2006), and DeMarzo and Fishman (2007), which study working capital dynamics in the relationship between a creditor and an entrepreneur. Without manager-worker collusion, we prove that creditor and worker essentially act as a single party. Consequently, our Lemma 1 implies that our productivity dynamics are closely related to the dynamics in Thomas and Worrall (1994) and Albuquerque and Hopenhayn (2004), with effort serving the role of working capital from those papers. The fact that dynamics arise from effort significantly changes the implications of these results, since changes in the manager-worker relationship manifest in productivity and wages rather than capital. Since the manager maintains relationships with two parties, the structure of her payments to one party affect the other relationship, leading to wage, debt, and productivity dynamics. With manager-worker collusion, liquidation and wage dynamics are less related to the dynamics in these papers.

A growing literature studies how agency problems interact with financial constraints (Pagano and Vulpin (2008)). Like us, Hennessy and Livdan (2009) and Fahn et al. (2018) consider how borrowing constrains relational contracts, but both focus on stationary rather than dynamic equilibria. Michelacci and Quadrini (2009) considers wage dynamics that arise from financial constraints. Unlike that paper, which considers how a firms might essentially “borrow money” by backloading worker pay in order to repay debts, we focus on how a firm motivates workers in a setting where it can commit to neither incentive pay nor credit repayments. Our analysis consequently focuses on both productivity and wage dynamics, and we study different theoretical tradeoffs and empirical implications. Li and Matouschek (2013), Englmaier and Fahn (2017), and Fuchs et al. (2017) consider liquidity constraints

or investments in relational contracts, but unlike our setting, these papers do not study the interaction between agency and credit relationships.

2 Model

Consider an infinite-horizon game with a manager (“she”), a creditor (“it”), and a worker (“he”) who share a common discount factor $\delta \in [0, 1)$ and interact in periods $t \in \{0, 1, 2, \dots\}$. The manager needs a loan of size $L > 0$ to start a project. To secure this funding, she offers a contract to the creditor that specifies a sequence of liquidation probabilities $l(\cdot) \in [0, 1]$ that can depend on the history of payments to the creditor. The payment to the creditor in period t is denoted r_t , with corresponding contractible history $h_c^t = (r_0, \dots, r_t)$.

The creditor accepts or rejects this formal contract. If it accepts, then it pays L , the project is funded, and the game continues; otherwise, the game ends and players earn 0. This decision is the creditor’s only action in the game, though the resulting contract $l(\cdot)$ determines the continuation game played by the manager and the worker.

If the project is funded, then the manager and the worker play a repeated game. All variables are publicly observed, but the only contractible variables are repayments to the creditor (r_t) and a public randomization device realized at every stage of each period. In each period, the stage game is:

1. The worker chooses effort $a_t \in \mathbb{R}_+$.
2. A state of the world $\theta_t \in \{0, 1\}$ is realized, with $\Pr\{\theta_t = 1\} = p$.
3. Output $y_t = \theta_t a_t$ is realized.
4. The manager pays $b_t \geq 0$ and $r_t \geq 0$ to the worker and creditor, respectively, where $b_t + r_t \leq y_t$.
5. The project is liquidated with probability $l_t \equiv l(h_c^t)$.

If the project has not yet been liquidated, then the manager’s and worker’s period- t payoffs are $\pi_t = y_t - b_t - r_t$ and $u_t = b_t - c(a_t)$, respectively, where $c(\cdot)$ is the worker’s cost of effort. We assume that $c(\cdot)$ is non-negative, strictly increasing, and strictly convex, with $c(0) = c'(0) = 0$ and $\lim_{a \rightarrow \infty} c'(a) = \infty$. The creditor earns r_t in period t . Following liquidation, the game

ends and all players earn 0. The manager's, worker's, and creditor's normalized discounted continuation payoffs in period t are $\Pi_t = \sum_{t'=t}^{\infty} \delta^{t'-t}(1-\delta)\pi_{t'}$, $U_t = \sum_{t'=t}^{\infty} \delta^{t'-t}(1-\delta)u_{t'}$, and $K_t = \sum_{t'=t}^{\infty} \delta^{t'-t}(1-\delta)r_{t'}$, respectively.

Let **first-best effort** a^{FB} satisfy $c'(a^{FB}) = p$. We assume throughout that $pa^{FB} - c(a^{FB}) > L$, so that the project has strictly positive net-present value if the worker exerts first-best effort. Define $h^t \in \mathcal{H}^t$ as a history in period t . We consider **profit-maximizing** Subgame Perfect Equilibria (SPE), which maximize the manager's *ex ante* expected payoff among SPE.

It is worth spending a moment on the central contracting friction in this model, which is that the manager can refuse to pay r_t or b_t and suffer no worse punishment than losing her continuation payoff. As in the literature on relational contracts, we model this situation by assuming that the formal contract can make neither payments nor liquidation conditional on y_t , which means that the manager can always renege on promised payments. In practice, while many financial instruments condition on realized profits, managers typically face opportunities to divert those profits in non-contractible ways, for instance by stealing them directly or diverting them to privately beneficial but unprofitable ventures (see, e.g., Hart (1995); Holmstrom and Tirole (1997); Albuquerque and Hopenhayn (2004); and DeMarzo and Fishman (2007)). Our formulation is equivalent to one in which *reported* profits are contractible but the manager can mis-report profits and pocket the difference.⁴

Several features of the model are convenient but inessential for our basic intuition. For instance, we assume the manager keeps the entire output y_t if she reneges on equilibrium payments, but our intuition continues to hold if she consumes only a (strictly positive) fraction of output after renegeing. Further, players earn 0 upon liquidation, but allowing a positive scrap value would not substantively change the analysis. We assume that the creditor observes everything in the repeated game, but our results do not depend on this assumption because the creditor takes no actions in the repeated game.

Other assumptions potentially affect the central intuition and so deserve further comment. First, apart from the initial loan, the manager can neither borrow nor save; we discuss this restriction further in Section 6. Second, effort is observable, which simplifies equilibrium

⁴For example, we could assume that reported profit \hat{y}_t is contractible but the manager can report $\hat{y}_t < y_t$ and effectively steal $y_t - \hat{y}_t$. If payments can condition on \hat{y}_t but not y_t and moreover must satisfy $b_t + r_t \leq \hat{y}_t$, then our analysis would continue to hold.

incentives in order to more starkly highlight how financing constraints lead to effort dynamics. Finally, we assume that liquidation $l(\cdot)$ can condition on repayments to the creditor but not on payments to the worker, which is irrelevant in Section 3 but is important in Section 4 because it allows the manager and worker to “secretly” make side payments to one another while colluding.

3 Financing Constraints and Productivity

This section analyzes productivity and payment dynamics in profit-maximizing equilibria. To do so, we formulate the equilibrium payoff frontier as a dynamic program, consider two benchmarks that highlight the role of the manager’s commitment problem, and then prove our main results.

The worker or the creditor can each punish the manager equally harshly following a deviation, the former by choosing $a_t = 0$ in each period and the latter by liquidating the firm. While either of these punishments min-max the manager, the worker can condition his punishment on the realization of y_t and thereby punish the manager exactly when she reneges on a payment she has the money to make. Consequently, it is efficient for the worker to punish *any* deviation by the manager, including deviations in r_t . We can therefore set $l(\cdot) \equiv 0$ and assume that the manager earns 0 following any deviation.

Given these punishments, we can write the dynamic program for a profit-maximizing equilibrium. Let

$$E = \{(U, \Pi) | \exists K \in \mathbb{R} \text{ s.t. } \exists \text{ SPE with worker, manager, and creditor payoffs } (U, \Pi, K)\}$$

be the set of the worker’s and the manager’s equilibrium continuation payoffs. Given $(U, \Pi) \in E$, define $K(U, \Pi)$ as the creditor’s maximum continuation payoff if the worker and manager earn U and Π , respectively. In a given period t , denote $b \geq 0$ and $r \geq 0$ as the manager’s payments to the worker and creditor if $\theta_t = 1$ and note that $b_t = r_t = 0$ if $\theta_t = 0$. Let (U_L, Π_L) and (U_H, Π_H) be the worker’s and manager’s continuation surpluses from period $t + 1$ onwards if $\theta_t = 0$ or $\theta_t = 1$, respectively.

Now, consider the constraints imposed by an equilibrium in which the manager and worker earn expected payoffs Π and U , respectively. First, play must satisfy **promise-**

keeping constraints so that the worker and manager actually earn these payoffs,

$$U = (1 - \delta)(pb - c(a)) + \delta(pU_H + (1 - p)U_L) \text{ and} \quad (\text{PK-A})$$

$$\Pi = (1 - \delta)p(a - b - r) + \delta(p\Pi_H + (1 - p)\Pi_L). \quad (\text{PK-P})$$

Second, the worker must be willing to choose the equilibrium effort level $a \geq 0$. He earns U from following the equilibrium and no more than 0 from deviating, so this **incentive constraint** is simply

$$U \geq 0. \quad (\text{IC})$$

Third, if $\theta = 1$, the manager must be willing to pay b and r . Since the manager earns 0 continuation profit if she deviates, she pays r and b if and only if these payments satisfy the **dynamic enforcement constraint**

$$\delta\Pi_H \geq (1 - \delta)(b + r). \quad (\text{DE})$$

Fourth, since $y = a$ if $\theta = 1$, r and b must satisfy the following **limited liability constraints**:

$$r \geq 0; \quad b \geq 0; \quad r + b \leq a. \quad (\text{LL})$$

Finally, continuation payoffs must be attainable in equilibrium,

$$(U_H, \Pi_H), (U_L, \Pi_L) \in E. \quad (\text{CE})$$

Given (U, Π) , actions and continuation payoffs in the equilibrium that maximizes the creditor's payoff solve

$$\begin{aligned} K(U, \Pi) \equiv & \max_{r, b, a, U_H, U_L, \Pi_H, \Pi_L} (1 - \delta)pr + \delta(pK(U_H, \Pi_H) + (1 - p)K(U_L, \Pi_L)) \quad (1) \\ & \text{subject to} \quad (\text{PK-A}), (\text{PK-P}), (\text{IC}), (\text{DE}), (\text{LL}), (\text{CE}). \end{aligned}$$

The project is funded if continuation utilities at the start of the game, $(U_0, \Pi_0) \in E$, satisfy $K(U_0, \Pi_0) \geq L$. Characterizing equilibrium dynamics therefore amounts to characterizing

$K(\cdot, \cdot)$, which is the equilibrium payoff frontier of this game.

We consider two benchmarks before analyzing $K(\cdot, \cdot)$. The first benchmark removes (DE) as a constraint, which eliminates the manager's commitment problem entirely. In the **commitment game**, the formal contract specifies payments b_t and r_t as a function of the history of realized outputs, $\{y_{t'}\}_{t'=0}^t$. The manager has no choice but to make these formally-contracted payments, which therefore do not need to satisfy (DE). Consequently, we show that profit-maximizing equilibria attain first-best.

Proposition 1 *In any profit-maximizing equilibrium of the commitment game, the project is funded and $a_t = a^{FB}$ for all $t \geq 0$.*

Proof: See Appendix A.

With output-contingent formal contracts, the manager can commit to pay $b = c(a)$ to exactly compensate the worker for his effort costs. She can also commit to repay the creditor any portion of the resulting proceeds from the (positive net present value) project.

Our second benchmark removes (IC) as a constraint by analyzing a setting in which the worker is essentially passive. In the **bilateral game**, the worker chooses a_t , which the manager can either accept or reject. If she rejects, then effort and output equal 0 in that period; if she accepts, then effort equals a_t and the manager—but *not* the worker—incurs cost $c(a_t)$. This benchmark essentially eliminates the need to motivate the worker, so we can set $b_t = 0$ in each period. As in the baseline game, however, the worker can still punish the manager following a deviation by choosing $a_t = 0$ in subsequent periods.

The manager must earn a strictly positive payoff to deter her from renegeing on the creditor. Therefore, she might not be able to credibly promise to repay the creditor even though the project has positive net-present value. If the project is funded, however, then we show that effort equals first-best in each period.

Proposition 2 *In any profit-maximizing equilibrium of the bilateral game, the project is funded if and only if $L \leq \frac{\delta p}{1-\delta+\delta p} (pa^{FB} - c(a^{FB}))$. If the project is funded, then $a_t = a^{FB}$ for all $t \geq 0$.*

Proof: See Appendix A.

Suppose the project is funded in the bilateral game. We can set $b_t = 0$ in each period t , in which case the manager earns $p_{at} - c(a_t) - r_t$. Fixing r_t , increasing a_t for $a_t < a^{FB}$ both increases the manager's payoff and relaxes all constraints. Any profit-maximizing equilibrium therefore implements first-best effort in each period. However, the manager is willing to repay the creditor only if she earns a strictly positive payoff, which creates a wedge between the value of the project and the amount that the creditor can be repaid in equilibrium.

Both of these benchmarks feature stationary profit-maximizing equilibria. In contrast, profit-maximizing equilibria of the full game exhibit substantial effort and payment dynamics. The first step in characterizing these dynamics is to simplify the dynamic programming problem. We prove that the total surplus that is promised to creditor and worker, $K(U, \Pi) + U$, depends on Π but not on how that surplus is split between those two players. Define

$$\tilde{U}(\Pi) = \max \{U | (U, \Pi) \in E\}$$

as the worker's maximum equilibrium continuation utility given Π .

Lemma 1 *For any $(U, \Pi) \in E$,*

$$K(U, \Pi) + U = \tilde{U}(\Pi).$$

Proof: See Appendix A.

Lemma 1 says that the profit-maximizing equilibrium depends only on the sum of the worker's and creditor's continuation payoffs, which substantially simplifies the analysis of these equilibria. One implication of this result is that total surplus, $\Pi + U + K(U, \Pi)$, depends on the *aggregate* promised continuation payoff to the creditor and agent, $K(U, \Pi) + U$, but not on *which of those two parties* is promised this continuation payoff. To prove Lemma 1, we show that given a sum $K(U, \Pi) + U > 0$, we can always find a period in which we can either decrease the creditor's payment in order to increase the worker's payment by the same amount, or decrease the worker's payment without violating (IC) in order to increase the creditor's payment. Consequently, we can increase or decrease $U \geq 0$ without changing $K(U, \Pi) + U$. This argument builds on the intuition from Levin (2002), but unlike that paper, our result holds even though manager and worker are both liquidity constrained, and

it does *not* imply that profit-maximizing equilibria are stationary. Our next result shows that these equilibria typically entail dynamics.⁵

Given Lemma 1, we can characterize the equilibrium payoff frontier in terms of $\tilde{U}(\Pi)$. Define

$$a_{max} = \arg \max_{a \geq 0} \left\{ pa - c(a) | \delta(pa - c(a)) \geq \frac{(1-\delta)c(a)}{p} \right\} \quad (2)$$

as the maximum effort that can be attained in equilibrium if $K(U, \Pi)$.

Proposition 3 *Consider on-path play in period t of a profit-maximizing equilibrium, and let Π be the manager's continuation payoff at the start of that period. Then:*

1. $a_t = a^*(\Pi)$, where

$$a^*(\Pi) \equiv \min \left\{ a_{max}, \frac{(1-\delta(1-p))}{(1-\delta)p} \Pi \right\}.$$

2. If $a_t < a_{max}$, then $\Pi_L = \Pi < \Pi_H$, $a_{t+1} = a_t$ if $\theta_t = 0$, and $a_{t+1} > a_t$ if $\theta_t = 1$.

3. If $a_t = a_{max}$, then $a_{t'} = a_{max}$ for all $t' > t$. Consequently,

$$\lim_{t \rightarrow \infty} Pr\{a_t = a_{max}\} = 1.$$

Proof: See Appendix A.

Proposition 3 characterizes effort dynamics in profit-maximizing equilibria. If the project is funded, then $U_0 = 0$ and Π_0 satisfies $K(0, \Pi_0) = L$ in $t = 0$. From there, effort remains constant after $\theta_t = 0$ and increases after $\theta_t = 1$ until $a_t = a_{max}$, at which point it stays at this steady state.

To prove this result, note that $\tilde{U}(\Pi)$ represents the payoff frontier if the creditor earns 0 continuation payoff. Along this frontier, we show that either $a = a^{FB}$ or (DE) binds, since otherwise the manager could profitably pay the worker more in exchange for higher effort. Therefore, b is pinned down by (DE); in particular, if $\tilde{U}(\Pi) = 0$, then $b = c(a)/p$ and $a = a_{max}$. If the worker earns strictly positive continuation utility, then he must not be working as hard as he would be willing to work given his expected compensation. Consequently, total

⁵We can show that a version of Lemma 1 holds in more general settings with multiple workers and creditors, if effort is observable and players jointly punish deviations. Consequently, similar productivity dynamics arise in richer environments.

surplus $\tilde{U}(\Pi) + \Pi$ is increasing in Π , which means the manager prefers paying the worker more today in exchange for a higher continuation payoff whenever $\tilde{U}(\Pi_H) > 0$.

This intuition pins down b and Π_H , so characterizing $\tilde{U}(\Pi)$ reduces to characterizing Π_L . Note that Π_L and U_L enter only (PK-A) and (PK-P). Since $\tilde{U}(\Pi)$ is concave, we can show that $\tilde{U}(\Pi_L) + \Pi_L = \tilde{U}(\Pi) + \Pi$ in any profit-maximizing equilibrium, which implies $\Pi_L = \Pi$ and completes the characterization of $\tilde{U}(\Pi)$. The shape of the payoff frontier $K(U, \Pi)$ then follows from Lemma 1.

The most important implication of Proposition 3 for our purposes is that, unless L is small, effort starts below a_{max} and only slowly and stochastically increases to this steady state. That is, the manager's outstanding obligations to *both* the creditor and the worker, $K(U, \Pi) + U$, determines equilibrium effort. Productivity dynamics arise because the amount owed to the creditor changes over time, with corresponding changes in the worker's compensation.

Profit-maximizing equilibria also exhibits two inefficiencies that are more familiar. First, as in many relational contracting models (e.g., Levin (2003)), even steady-state effort a_{max} is below first-best unless players are quite patient. Second, as in many papers on financing constraints (e.g., Holmstrom and Tirole (1997)), positive net-present value projects might go unfunded for reasons similar to Proposition 2 (that is, $K(0, 0) = 0$).

We now turn to compensation dynamics. While many payment paths are consistent with a profit-maximizing equilibrium, our next result identifies two extremes that repay the creditor either as quickly or as slowly as possible.

Corollary 1 *The following payment paths are each part of a profit-maximizing equilibrium.*

1. Fastest repayment equilibrium: $r_t = y_t$ whenever $K(U_H, \Pi_H) > 0$. Once $K(U, \Pi) = 0$, $b_t = y_t$ whenever $U_H > 0$.
2. Slowest repayment equilibrium: whenever $K(0, \Pi_H) > 0$, $r_t = y_t - \frac{c(a_t)}{p}$ and $b_t = \frac{c(a_t)}{p}$, so that $U = 0$ in every period.

Proof: See Appendix A.

Corollary 1 follows from Lemma 1 and the proof of Proposition 3. Before productivity reaches its steady state, every profit-maximizing equilibrium entails the same *total* payment

to the creditor and worker; the only question is how this payment is split between the two parties. To repay the creditor as quickly as possible, the manager defers the worker's compensation until after the loan has been completely repaid. In contrast, the slowest repayment path exactly compensates the worker for his effort cost in each period.

If part of the worker's pay is deferred until after the creditor is repaid, as it is in the fastest repayment equilibrium, then $U > 0$ in the first period that satisfies $K(U, \Pi) = 0$. If $U_H > 0$ in that period as well, then $a < a_{max}$ in that period. That is, productivity dynamics (temporarily) persist even after the creditor has been repaid.

Lemma 1 shows that in a profit-maximizing equilibrium, the worker and the creditor essentially act as a single entity that loans the manager money and then exerts effort in each period. Consequently, the effort dynamics in Proposition 3 follow the working capital dynamics from Albuquerque and Hopenhayn (2004). There are three substantive differences. First, the worker cannot earn negative continuation utility, which implies that steady-state effort might remain inefficiently low, $a_{max} < a^{FB}$. Second, Albuquerque and Hopenhayn (2004) considers capital dynamics and so do not speak to either productivity or compensation dynamics. Finally, if borrowing leads to deferred worker compensation, then effort temporarily remains below the steady-state in our setting even after the creditor has been repaid. Section 5 explores the latter two differences empirically. The next section turns to a setting in which Lemma 1 does not hold, which gives rise to further dynamics, including the possibility of on-path liquidation.

4 Collusion Between the Manager and Worker

Section 3 assumes that the worker punishes the manager for renegeing on the creditor. In practice, the manager and worker might collude to undermine these coordinated punishments. The creditor deters collusion by threatening to liquidate the project if it is not repaid promptly. Liquidation cannot be made contingent on realized output, however, so the resulting equilibria entail liquidation as well as compensation dynamics.

This section analyzes equilibria that deter collusion. Our analysis focuses on a binary-effort version of the model: $a_t \in \{0, y\}$, where (with abuse of notation) $c(0) = 0$ and $c(y) = c$. To understand how we model manager-worker collusion, consider the following thought experiment. Suppose that *after* the creditor has agreed to loan money to the

principal, the principal can choose the continuation equilibrium that she and the worker will play. Then she optimally chooses an equilibrium in which the worker punishes her for renegeing on b_t , since that is the only way to induce the worker to exert effort. Since the creditor's loan is sunk at that point, however, the manager chooses an equilibrium that does *not* have the worker punishing the manager for renegeing on r_t . Consequently, the creditor cannot rely on the worker to punish on its behalf; it can guarantee repayment only by threatening liquidation.

Online Appendix C formalizes this thought experiment by introducing an equilibrium refinement that captures collusion. In that appendix, we show that collusion is deterred if the *sum* of the worker's and manager's continuation surpluses, minus the payment to the creditor, to be increasing in the shock θ . We call an equilibrium that satisfies this sufficient condition a **truth-telling equilibrium**.

Definition 1 A Subgame Perfect Equilibrium σ^* is a **truth-telling equilibrium** if at every on-path history h^t immediately before θ_t is realized,

$$E_{\sigma^*} [-(1-\delta)r_t + \delta(\Pi_{t+1} + U_{t+1}) | h^t, \theta_t = 1] \geq E_{\sigma^*} [\delta(\Pi_{t+1} + U_{t+1}) | h^t, \theta_t = 0]. \quad (3)$$

Online Appendix C proves that (3) is enough to deter collusion and discusses this condition further. Note that (3) is related to the truth-telling constraints that appear in many entrepreneur-creditor models (e.g., Clementi and Hopenhayn (2006); DeMarzo and Fishman (2007)), with the key difference that (3) depends on the sum of the worker's and the manager's payoffs rather than the payoff of a single player.

The recursive form of (3) is the following **truth-telling** constraint:

$$(1-\delta)r \leq \delta(\Pi_H + U_H - \Pi_L - U_L). \quad (\text{TT})$$

Let $K^T(U, \Pi)$ be the solution to (1) subject to (PK-A)-(CE) and (TT), given $a_t \in \{0, y\}$. Continuation payoffs always lie on this payoff frontier, so we can characterize equilibrium dynamics by characterizing $K^T(\cdot)$.

Proposition 4 Suppose

$$\frac{c}{p} \leq \frac{\delta}{1-\delta}(py - c). \quad (4)$$

Then:

1. Fix $\Pi \geq \frac{p(1-\delta)}{1-(1-p)\delta}y$. If $U \in \mathbb{R}_+$ is such that $K^T(U, \Pi) > 0$, then for any $U' > U$ such that (U', Π) can be attained in a truth-telling equilibrium, $K^T(U, \Pi) + U < K^T(U', \Pi) + U'$.
2. If $K^T(U_H, \Pi_H) > 0$, $b = 0$ and $r = y$.
3. If $K^T(U, \Pi) > 0$, then liquidation occurs with positive probability in the continuation equilibrium.

Proof: See Appendix A.

The condition (4) ensures that the worker could be motivated to exert effort if the manager did not need to borrow money. The first part of Proposition 4 says that Lemma 1 does not hold for truth-telling equilibria. Since repayments to the creditor must satisfy (TT), those repayments decrease total continuation surplus in a way that promises to the worker do not. The second part of Proposition 4 says that to mitigate the negative effects of this additional constraint, the manager initially devotes all output to repaying the creditor. During these periods, the worker is paid nothing and so is motivated only by the promise of future compensation. The final part of the proposition identifies why outstanding financial obligations are uniquely costly: so long as the creditor has not yet been fully repaid, inefficient liquidation occurs on the equilibrium path. In short: the creditor threatens liquidation to satisfy (TT). The manager minimizes the probability of liquidation by frontloading loan repayment and backloading worker compensation. Total surplus and the worker's continuation payoff therefore depend on current and past debts.

The proof of Proposition 4 is significantly more challenging than that of Proposition 3 because Lemma 1 does not hold in a truth-telling equilibrium. Consequently, equilibrium dynamics depend on the promised utilities to *each of* the creditor and the worker rather than on the *sum* of those utilities. We first argue that liquidating the project with positive probability is at least as efficient as setting $a = 0$, which means that we can set $a = y$ so long as the project has not yet been liquidated. Then we show that (TT) holds with $r > 0$ only if liquidation might occur after (a sequence of) $\theta = 0$ realizations, which means that liquidation occurs on-path so long as the creditor has not been fully repaid. The manager pays the creditor before the worker in order to minimize the probability of liquidation, which

implies that the worker’s continuation utility is history-dependent and strictly positive in the first period after the debt is repaid.

Proposition 4 says that *every* profit-maximizing truth-telling equilibrium entails deferred worker compensation, which strengthens Corollary 1’s result that *there exist* profit-maximizing equilibria with deferred compensation. Unlike Proposition 3, the firm is sometimes liquidated in this equilibrium, which means that play eventually reaches one of two steady states: either the loan is repaid and continuation surplus converges to $\frac{\delta}{1-\delta}(py - c)$, or the firm is liquidated for failing to repay its debts.

Binary effort simplifies the analysis but is not necessary for our basic argument. In Online Appendix D, we characterize truth-telling equilibria in a setting with continuous effort. As in Proposition 4, we show that in the setting with continuous effort, the manager still optimally backloads worker pay whenever (TT) binds. Consequently, just as in the setting with binary effort, debt has a lingering effect on the worker’s continuation utility and hence on his effort.

5 Empirical Results

We can summarize the basic mechanism that drives both Propositions 3 and 4 in the following way: an indebted manager prioritizes repaying those debts, which means that she cannot credibly promise much compensation to workers, which leads to less effort. The manager might also defer worker compensation, which leads to lower effort in the future as well.

This section documents correlations that are consistent with (though not conclusive of) this story. Controlling for firm and year or industry-year fixed effects, we first show that increases in a firm’s leverage are correlated with decreases in its (i) *current* compensation, (ii) *current* productivity, and (iii) *future* productivity. We then analyze heterogeneity in these effects. Consistent with our mechanism, we show that these correlations are strongest in (i) industries that rely heavily on labor inputs and (ii) countries where formal contract enforcement is relatively expensive.

Our analysis uses data from Amadeus, which includes yearly financial statements, measures of input, and revenue for a panel of European firms. Our sample covers 16 countries and almost 25 thousand public and private manufacturing firms, with up to 10 years of data per firm between 1985-2017. We measure borrowing by the ratio of non-current liabilities to total assets (book leverage); unlike market leverage, this measure of debt does not directly

depend on market beliefs about current or future profits. We proxy for productivity using several measures of revenue total factor productivity (TFP-R). Our simplest specification, which measures TFP-R as the residual from an OLS regression of revenue on capital and labor, suffers from endogeneity concerns because firms' input choices potentially respond to their unobserved productivity shocks. Therefore, we also calculate TFP-R using methods from Levinsohn and Petrin (2003) and Gandhi et al. (2017), both of which use a firm's intermediate input choices to address this concern. We use output revenue, fixed assets, total wage bill, and the cost of materials to proxy for output, capital, labor, and intermediate inputs, respectively. Our analysis restricts attention to industry-country pairs with at least 1,000 observations. Appendix B details our methodology.

Our first results regress changes in either wages or productivity on changes in both current- and previous-year leverage:

$$\Delta Dep_{i,t} = \beta_1 \Delta Lev_{i,t} + \beta_2 \Delta Lev_{i,t-1} + \beta_3 X_{i,t} + \mu_i + \psi_t + \epsilon_{i,t}. \quad (5)$$

Here, $\Delta Dep_{i,t}$ is the change from year $t - 1$ to year t in either TFP-R or wages, where wages are measured by total compensation costs divided by number of workers. Similarly, $\Delta Lev_{i,t}$ is the change in financial leverage from $t - 1$ to t . We include both firm and year fixed effects, so our regression exploits within-firm changes in productivity and leverage while controlling for year-specific aggregate shocks. We also run an alternative specification that includes industry-year fixed effects in order to control for year-specific shocks to industry-level productivity. Additional controls $X_{i,t}$ include changes in number of employees and in total fixed assets. Our main results weight observations equally; we find similar results if we weight by number of employees, total assets, or gross output, though $\Delta Lev_{i,t-1}$ is occasionally insignificant. Standard errors are clustered by NACE2 industry code.

Table 1 reports the results of (5) with wage changes as the dependent variable. After including either firm and year or industry-year fixed effects, we find that an increase in leverage is correlated with a decrease in contemporaneous wages. The coefficient on lagged leverage is negative but not significant. These results are broadly consistent with our mechanism, which says that compensation (as well as effort) tend to be lower when a firm's outstanding financial obligations are high. It is also consistent with Benmelech et al. (2012), which finds that firms in financial distress negotiate substantially less generous labor contracts, and Matsa (2010), which argues that firms take on debt in order to improve their bargaining

Table 1: Changes in Average Wage and Changes in Leverage

| | (1) $\Delta L(\text{Wage})$ | (2) $\Delta L(\text{Wage})$ |
|--------------------------------|--------------------------------|--------------------------------|
| $\Delta \text{Leverage}$ | -0.109 (0.0144) | -0.0832 (0.0164) |
| $\Delta \text{Leverage (t-1)}$ | -0.0180 (0.0125) | -0.0183 (0.0104) |
| N | 127703 | 127703 |
| Clustering | Industry | Industry |
| Firm FE | Yes | - |
| Year FE | Yes | - |
| Industry X Year FE | - | Yes |

Additional controls include total fixed assets and number of employees. Leverage is measured as the ratio of non-current liabilities to total assets (book value). Dependent variable is change in log of average wage which is calculated as the ratio of total compensation costs and number of employees. Standard errors are clustered on the industry level and shown in the parentheses.

position versus organized labor.⁶

Table 2 gives the analogous results for productivity. We find that an increase in firm leverage in year t or $t-1$ is correlated with a decrease in year- t productivity. This relationship is highly significant across measures of TFP-R and economically substantial: a one standard deviation increase in contemporary leverage is correlated with a decrease in TFP-R equal to 9-15% of the median within-firm standard deviation (see Appendix B for details on these calculations). The coefficient on the previous year's leverage change is smaller but remains significant across specifications.

Our productivity regressions should be interpreted with extra caution. In particular, a similar relationship between debt and *contemporaneous* productivity would arise if firms that suffer negative idiosyncratic productivity shocks have trouble funding operations and so take on more debt. The (less robust) correlation between *last year's* leverage change and *this year's* productivity change is less susceptible to this alternative explanation. Indeed,

⁶Note that in our model, lower *wages* do not necessarily imply that the worker earns lower *utility*, since poorly compensated workers exert less effort. Therefore, our analysis is consistent with the idea that firms with higher leverage must compensate workers for bankruptcy risk.

Table 2: Changes in TFP and Changes in Leverage

| | (1) $\Delta\text{TFP-}$ OLS | (2) $\Delta\text{TFP-}$ OLS | (3) $\Delta\text{TFP-LP}$ | (4) $\Delta\text{TFP-LP}$ | (5) $\Delta\text{TFP-}$ GNR | (6) $\Delta\text{TFP-}$ GNR |
|----------------------------------|-----------------------------------|-----------------------------------|------------------------------|------------------------------|-----------------------------------|-----------------------------------|
| $\Delta\text{Leverage}$ | -0.432 (0.0472) | -0.417 (0.0412) | -0.160 (0.0229) | -0.150 (0.0203) | -0.475 (0.0749) | -0.409 (0.0650) |
| $\Delta\text{Leverage}$ (t-1) | -0.101 (0.0200) | -0.0858 (0.0137) | -0.0449 (0.0187) | -0.0452 (0.0183) | -0.0903 (0.0211) | -0.0650 (0.0108) |
| N | 127703 | 127703 | 127703 | 127703 | 127703 | 127703 |
| SD of Dep Var | 0.12 | 0.12 | 0.08 | 0.08 | 0.16 | 0.16 |
| SD of ΔLev | 0.044 | 0.044 | 0.044 | 0.044 | 0.044 | 0.044 |
| Clustering | Industry | Industry | Industry | Industry | Industry | Industry |
| Firm FE | Yes | - | Yes | - | Yes | - |
| Year FE | Yes | - | Yes | - | Yes | - |
| Industry \times Year FE | - | Yes | - | Yes | - | Yes |

Additional controls include total fixed assets and number of employees. Leverage is measured as the ratio of non-current liabilities to total assets (book value). Dependent variable is change in TFP-R which is calculated based on OLS specification (columns 1 and 2), Levinsohn and Petrin (2003) method (LP, columns 3 and 4) and Gandhi et al. (2017) method (GNR, columns 5 and 6). In the bottom rows, standard deviation of dependent variable and of change in leverage is a within-firm value and median across all firms is presented. Standard errors are clustered on the industry level and shown in the parentheses.

Table 3: Effect for Labor Intensive Industries

| | (1) $\Delta L(\text{Wage})$ | (2) $\Delta \text{TFP-OLS}$ | (3) $\Delta \text{TFP-LP}$ | (4) $\Delta \text{TFP-GNR}$ |
|---|--------------------------------|--------------------------------|-------------------------------|--------------------------------|
| $\Delta \text{Leverage}$ | -0.0366 (0.0165) | -0.352 (0.0529) | -0.0846 (0.0279) | -0.290 (0.0612) |
| $\Delta \text{Leverage} \times \text{Labor}$ Intensive | -0.110 (0.0274) | -0.136 (0.0803) | -0.129 (0.0466) | -0.313 (0.108) |
| $\Delta \text{Leverage (t-1)}$ | -0.0498 (0.0141) | -0.103 (0.0214) | -0.0283 (0.0122) | -0.0884 (0.0196) |
| $\Delta \text{Leverage (t-1)} \times \text{Labor}$ Intensive | 0.0085 (0.0211) | 0.00148 (0.0442) | -0.0289 (0.0353) | -0.00601 (0.0390) |
| N | 127703 | 127703 | 127703 | 127703 |
| Clustering | Industry | Industry | Industry | Industry |
| Firm FE | Yes | Yes | Yes | Yes |
| Year FE | Yes | Yes | Yes | Yes |
| Industry \times Year FE | - | - | - | - |

Additional controls include total fixed assets and number of employees. Leverage is measured as the ratio of non-current liabilities to total assets (book value). Variable Labor Intensive takes value 1 if industry share of labor input in production function is above country-wide median, and 0 otherwise. Dependent variable is change in log of average wage which is calculated as the ratio of total compensation costs and number of employees; and change of TFP-R which is calculated based on OLS specification (column 2), Levinsohn and Petrin (2003) method (LP, column 3) and Gandhi et al. (2017) method (GNR, column 4). Standard errors are clustered on the industry level and shown in the parentheses.

the alternative explanation could account for this correlation only if changes in TFP-R were persistent, so that low TFP-R leads to more borrowing in both this year and the next. To address this possibility, Appendix B presents a regression that includes lagged productivity changes, $\Delta \text{TFP}_{i,t-1}$, as a regressor. Our results remain significant and many of the coefficients have larger magnitudes in this alternative specification.⁷

Our mechanism operates through worker incentives, which suggests that leverage should have a more pronounced effect on wages and productivity in industries that rely heavily

⁷Leverage might also be related to productivity shocks through changes in the firm's equity valuation, for instance because the firm's market value decreases when it receives a negative productivity shock. Such a relationship would generate a negative relationship between productivity and *market* leverage. However, we use *book* rather than market leverage, which alleviates (but does not eliminate) this concern because current profits are only a small part of total assets.

on labor inputs. We investigate this source of heterogeneity by interacting leverage with a country-industry proxy for labor intensity, as measured by an indicator that equals 1 whenever the OLS-based coefficient of labor in the production function for a given industry is above the country-wide median.⁸ Table 3 offers suggestive evidence in favor of this implication. Column 1 includes this interaction in the version of (5) where wages are the dependent variable, with the result that in more labor-intensive industries, an increase in leverage is correlated with a significantly larger decrease in wages. Columns 2, 3, and 4 run (5) using our three measures of productivity as dependent variables; the coefficient on the interaction between productivity and labor intensity is significant and negative in 2 of 3 specifications (the p-value for the OLS coefficient is 0.11). Coefficients on the lagged interaction are typically negative but not significant.

Finally, we explore how these relationships vary with the strength of a country's contracting institutions. The central friction in our model is a commitment problem: the manager can renege on output-contingent payments. This commitment problem is presumably more severe, and the relationship between leverage, wages, and productivity is therefore stronger, if formal enforcement mechanisms are weak. To investigate this claim, Table 4 runs a version of (5) that includes interactions between current and lagged leverage and a country-level measure of the cost of contract enforcement from the World Bank's Doing Business survey.⁹ This measure is used by Fahn et al. (2017) as a proxy for the prevalence of relational contracts. We find suggestive evidence that in countries with a high cost of contract enforcement, leverage changes are more strongly correlated with both wage changes (Column 1) and our three measures of productivity changes (Columns 2, 3, and 4). The coefficient on the contemporaneous interaction is typically significant, while the coefficient on the lagged interaction is typically not significant.

In our model, the worker's effort affects contemporaneous profits but does not have a persistent effect on firm profitability. In practice, however, credibility problems could also constrain hiring, retention, firm-specific human capital investments, and other choices that change a firm's long-term growth. Matsa (2017) has a recent review of how a firm's financial structure impacts its employment practices. Particularly related are recent empirical papers

⁸We do not need to include this indicator as a separate regressor because it is absorbed by firm fixed effects.

⁹Our main results use a binary indicator of above-median cost of contract enforcement but the analysis is robust to using continuous measure of the cost of enforcement.

Table 4: Effect for Countries with Weak Contract Enforcement

| | (1) $\Delta L(\text{Wage})$ | (2) $\Delta \text{TFP-OLS}$ | (3) $\Delta \text{TFP-LP}$ | (4) $\Delta \text{TFP-GNR}$ |
|---|--------------------------------|--------------------------------|-------------------------------|--------------------------------|
| $\Delta \text{Leverage}$ | 0.0016 (0.0207) | -0.296 (0.028) | -0.117 (0.034) | -0.292 (0.042) |
| $\Delta \text{Leverage} \times \text{Costly Enforcement}$ | -0.186 (0.036) | -0.248 (0.077) | -0.0784 (0.0857) | -0.332 (0.098) |
| $\Delta \text{Leverage (t-1)}$ | -0.0468 (0.0161) | -0.0894 (0.0214) | -0.0533 (0.0290) | -0.118 (0.034) |
| $\Delta \text{Leverage (t-1)} \times \text{Costly Enforcement}$ | 0.0017 (0.0151) | -0.0245 (0.0270) | 0.0137 (0.0306) | 0.046 (0.036) |
| N | 127604 | 127604 | 127604 | 127604 |
| Clustering | Industry | Industry | Industry | Industry |
| Firm FE | Yes | Yes | Yes | Yes |
| Year FE | Yes | Yes | Yes | Yes |
| Industry \times Year FE | - | - | - | - |

Additional controls include total fixed assets and number of employees. Leverage is measured as the ratio of non-current liabilities to total assets (book value). Variable Costly Enforcement takes value 1 if country-level measure of cost of contract enforcement (from Doing Business Survey) is above median, and 0 otherwise. Dependent variable is change in log of average wage which is calculated as the ratio of total compensation costs and number of employees; and change of TFP-R which is calculated based on OLS specification (column 2), Levinsohn and Petrin (2003) method (LP, column 3) and Gandhi et al. (2017) method (GNR, column 4). Standard errors are clustered on the industry level and shown in the parentheses.

on the consequences of debt for employees' hiring and retention (Brown and Matsa (2016); Baghai et al. (2016)). Relative to those papers, our theory suggests that debt might decrease productivity even if firms are far from bankruptcy. Our analysis also complements papers that study credit access and productivity, such as Minaresi and Pierri (2018). We argue that while the availability of credit might have a positive net effect on productivity, the act of borrowing itself can have lingering negative effects on effort.

6 Discussion

This section informally discusses several extensions and concludes.

Saving and borrowing: Apart from the initial loan, the manager neither borrows nor saves in our model. If the manager could borrow additional funds from the creditor, then she could pay the worker more, which would decrease the worker's promised continuation utility at the cost of increasing the creditor's promised continuation utility. Without collusion, Lemma 1 suggests the *sum* of these utilities determines productivity, so that the manager would not benefit from further borrowing. With collusion, Proposition 4 suggests that further borrowing would increase the probability of liquidation and would therefore be strictly inefficient. This intuition suggests that allowing further borrowing would not significantly change our results.

If the manager can save money and access those savings after she reneges, then she would still like to repay her obligations as quickly as possible, so we believe that savings would be irrelevant. In contrast, if the creditor or worker could prevent the manager from accessing savings, then accumulated savings could serve as a bond to deter the manager from deviating, which *would* potentially change equilibrium dynamics.

Investments with variable scale: In some settings, the manager chooses the scale of her investment. Our model suggests two opposing distortions in the optimal investment scale. First, borrowing decreases effort, which should push the manager to pursue smaller, less expensive projects. On the other hand, investing in certain kinds of projects can ease commitment problems, particularly if a larger project implies that the manager has more to lose following a deviation (Klein and Leffler (1981); Halac (2015); Englmaier and Fahn

(2017)). We can construct examples in which either of these two forces dominate, leading to either “insufficient” or “excessive” investment relative to first-best.

Conclusion: Our model highlights how borrowing influences the credibility of a firm’s promised incentives and thereby impacts effort, which is consistent with our empirical findings that changes in debt are negatively correlated with changes in wages and productivity. More broadly, our model demonstrates how a firm’s financial obligations can constrain its organizational structure. Understanding these spillovers is crucial for understanding firm productivity, profitability, and growth.

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A Omitted Proofs

A.1 Proof of Proposition 1

In the commitment game, suppose the manager offers a contract with $r_t(y_t) = \frac{L}{p}1\{y_t \geq \frac{L}{p}\}$, $b_t(y_t) = \frac{c(a^{FB})}{p}1\{y_t = a^{FB}\}$, and $l_t = 0$ in each $t \geq 0$. The creditor earns $pr_t(y_t) = L$ from this contract, while the worker earns $p\frac{c(a^{FB})}{p} - c(a^{FB}) = 0$ from choose a^{FB} and no more than 0 from choosing any other effort. Therefore, this contract induces the creditor to fund the project and the worker to choose $a_t = a^{FB}$ in each period. The manager earns $pa^{FB} - c(a^{FB}) - L > 0$, which is the maximum attainable surplus in any equilibrium. Therefore, this contract is profit-maximizing and any profit-maximizing contract must fund the project and induce a^{FB} in each period. ■

A.2 Proof of Proposition 2

In the unitary firm game, consider the following strategy: the manager offers a contract with $l_t = 0$ in each t , which the creditor accepts. In each t on the equilibrium path, $a_t = a^{FB}$, the manager accepts this effort, $b_t = 0$, and $r_t = 1\{y_t = a^{FB}\}\frac{L}{p}$. Following any deviation, the worker chooses $a_t = 0$ and the manager pays $b_t = r_t = 0$.

If $L \leq \frac{\delta p}{1-\delta+\delta p}(pa^{FB} - c(a^{FB}))$, then this strategy is an equilibrium. The manager earns $pa^{FB} - c(a^{FB}) - L > 0$ and accepts $a_t = a^{FB}$. She is willing to pay $r_t = \frac{L}{p}$ so long as

$$(1 - \delta)\frac{L}{p} \leq \delta(pa^{FB} - c(a^{FB}) - L)$$

which is implied by $L \leq \frac{\delta p}{1-\delta+\delta p}(pa^{FB} - c(a^{FB}))$. She is able to pay $r_t = \frac{L}{p}$ because $\frac{L}{p} \leq a^{FB} - \frac{c(a^{FB})}{p} < a^{FB}$. The manager earns the maximum attainable profit $pa^{FB} - c(a^{FB}) - L > 0$ in this equilibrium, so if $L \leq \frac{\delta p}{1-\delta+\delta p}(pa^{FB} - c(a^{FB}))$, then any profit-maximizing equilibrium entails a funded project and $a_t = a^{FB}$ in each period t on-path.

In any equilibrium, $\Pi_t \leq pa^{FB} - c(a^{FB}) - K_t$ because $U_t \geq 0$. Therefore, (DE) requires that at any h^t immediately following $\theta_t = 1$,

$$(1 - \delta)r_t \leq \delta(pa^{FB} - c(a^{FB}) - E[K_{t+1}|h^t]). \quad (6)$$

For each $t \geq 0$, define

$$\mathcal{H}(t) \equiv \{h^t | \theta_t = 1, \theta_{t'} = 0 \forall t' < t\}$$

as the set of histories such that $\theta_t = 1$ for the first time in period t . Then the project is funded only if

$$\sum_{t=0}^{\infty} \delta^t (1-p)^t p E[(1-\delta)r_t + \delta K_{t+1} | \mathcal{H}(t)] \geq L.$$

Applying (6) to this expression yields

$$\delta p \sum_{t=0}^{\infty} \delta^t (1-p)^t S^{FB} \geq L$$

or $\frac{\delta p}{1-\delta+\delta p} S^{FB} \geq L$. So the project cannot be funded if $L > \frac{\delta p}{1-\delta+\delta p} S^{FB}$. ■

A.3 Proof of Lemma 1

We first prove that $K(\tilde{U}(\Pi), \Pi) = 0$ for any $(U, \Pi) \in E$. Towards contradiction, suppose $K(\tilde{U}(\Pi), \Pi) > 0$ for some $(U, \Pi) \in E$. Then there exists some future period in which $r > 0$. In this period, consider the following perturbation: decrease r to $\tilde{r} = r - \epsilon$ and increase b to $\tilde{b} = b + \epsilon$. For sufficiently small $\epsilon > 0$, these perturbed payoffs continue to satisfy (DE) and (LL) in that and all previous periods because $r > 0$ and $r + b = \tilde{r} + \tilde{b}$, while (IC) is relaxed in that and all previous periods. But then $U < \tilde{U}(\Pi)$ and we obtain a contradiction. So $K(\tilde{U}(\Pi), \Pi) = 0$ for all $(U, \Pi) \in E$.

Next, we prove that $K(U, \Pi) = \tilde{U}(\Pi) - U$. If $U = \tilde{U}(\Pi)$, then this result holds by the previous argument. Note that if $\tilde{U}(\Pi) = 0$, then $U = \tilde{U}(\Pi)$. Suppose $0 \leq U < \tilde{U}(\Pi)$. Then we claim that $K(U, \Pi) > 0$. To prove this, consider the equilibrium in which the worker and manager earn $\tilde{U}(\Pi)$ and Π , respectively. We claim that there exists some period t and history h^t at the start of that period such that (i) $b_t > 0$ if demand is high in that period, (ii) $E[U_t | h^t] > 0$, and (iii) $E[U_{t'} | h^t] > 0$ for any $h^{t'}$ that precedes h^t .

To prove this, for each $\tau \geq 1$, define \mathcal{H}^τ as the set of histories such that $E[U | h^\tau] = 0$, but $E[U | h^t] > 0$ for all h^t that precedes h^τ . Define \mathcal{H}^∞ , with element $h^\infty \in \mathcal{H}^\infty$, as the set of infinite-horizon histories for which $E[U | h^\tau] > 0$ for every h^τ that precedes h^∞ . Then

$\mathcal{H}^\tau \cap \mathcal{H}^{\tau'} = \emptyset$ and $\bigcup_{\tau=0}^{\infty} \mathcal{H}^\tau = \mathcal{H}$, so the worker's payoff can be written

$$U = \sum_{\tau=0}^{\infty} \Pr_{\sigma^*}\{\mathcal{H}^\tau\} \left(E \left[(1-\delta) \sum_{t=0}^{\tau-1} \delta^t (b_t - c(a_t)) \mid \mathcal{H}^\tau \right] \right).$$

Since $U > 0$ and $c(a_t) \geq 0$, it cannot be that $b_t \equiv 0$ in this expression. That is, there must exist some τ , $h^\tau \in \mathcal{H}^\tau$, and h^t that precedes h^τ such that $b_t > 0$ with positive probability at h^t . By definition, $E[U_t|h^t] > 0$ and $E[U_{t'}|h^{t'}] > 0$ for any $h^{t'}$ that precedes h^t .

Consider decreasing $b_t > 0$ at this h^t and increasing r_t by the same amount. This perturbation satisfies (DE) and (LL) because $b_t > 0$ and $b_t + r_t$ is constant, while (IC) is slack at h^t and every predecessor history and so continues to hold for a sufficiently small perturbation. Since $K + U$ remains constant in this perturbation, which can be performed for any $U > 0$, $K(U, \Pi) > 0$ whenever $0 < U < \tilde{U}(\Pi)$. If $U = 0$, then $K(U, \Pi) > 0$, and we can decrease r in some period holding $b + r$ fixed to transfer utility from the creditor to the worker. Therefore, $K(U, \Pi) + U$ is constant in Π , and so $K(U, \Pi) = \tilde{U}(\Pi) - U$. ■

A.4 Proof of Proposition 3

By Lemma 1, $\tilde{U}(\Pi)$ characterizes equilibrium payoffs. Note that the public randomization device implies that $\tilde{U}(\Pi)$ is concave. Let Π_{max} be the maximum Π such that $(U, \Pi) \in E$, and note that when $U = \tilde{U}(\Pi)$, $r_t = 0$ in every period t on the equilibrium path.

Part 1: $\tilde{U}(\Pi) + \Pi$ is increasing in Π , and strictly so unless $\tilde{U}(\Pi) + \Pi = S^{FB}$. First, we claim that $\tilde{U}(\Pi) + \Pi$ is weakly increasing in Π .

For any Π such that $\tilde{U}(\Pi) > 0$, the proof of Lemma 1 implies that there exists some history h^t such that $b_t > 0$ with positive probability at h^t , $E[U_t|h^t] > 0$, and $E[U_{t'}|h^{t'}] > 0$ for any $h^{t'}$ that precedes h^t . Consider decreasing b_t . Both (DE) and (LL) are relaxed by this change, while (IC) continues to hold in every previous period because it was previously slack. So this perturbation is consistent with equilibrium, increases Π , decreases U , and holds total surplus fixed. Hence, total surplus $\tilde{U}(\Pi) + \Pi$ must be weakly increasing in Π .

Since $\tilde{U}(\cdot)$ is concave, there exists a (possibly corner) $\bar{\Pi}$ such that $\tilde{U}(\Pi) + \Pi$ is strictly increasing for $\Pi < \bar{\Pi}$. We argue that (DE) binds whenever $a < a^{FB}$. Suppose not; then we can perturb the equilibrium by increasing b and a so that $pb - c(a)$ remains constant. This

perturbation satisfies (IC) by construction, (DE) by assumption, and (LL) because $c'(a) < p$ for $a < a^{FB}$.

Now, consider two cases. First, suppose

$$(1 - \delta) \frac{c(a^{FB})}{p} \leq \delta(pa^{FB} - c(a^{FB})). \quad (7)$$

Then we claim that $\tilde{U}(\bar{\Pi}) + \bar{\Pi} = S^{FB}$. Consider the stationary strategy profile that sets $a = a^{FB}$ and $b = \frac{c(a^{FB})}{p}$. This strategy profile clearly satisfies (IC) and (LL), and also satisfies (DE) because (7) holds. Therefore, $(0, S^{FB}) \in E$ and hence $\tilde{U}(\bar{\Pi}) + \bar{\Pi} = S^{FB}$.

Suppose instead that (7) does not hold. Then we claim that $\tilde{U}(\bar{\Pi}) = 0$. Suppose towards contradiction that $\tilde{U}(\bar{\Pi}) > 0$. If $\Pi_L < \bar{\Pi}$, then we can perturb the equilibrium by increasing Π_L , which would strictly increase total surplus by definition of $\bar{\Pi}$ and continue to satisfy (IC) because $\tilde{U}(\bar{\Pi}) > 0$. Similarly if $\Pi_H < \bar{\Pi}$; hence, $\Pi_L, \Pi_H \geq \bar{\Pi}$. But then $\tilde{U}(\bar{\Pi}) + \bar{\Pi} = (p\bar{a} - c(\bar{a}))$, where \bar{a} is the effort induced in any equilibrium with $\Pi \geq \bar{\Pi}$ and $U = \tilde{U}(\Pi)$. Hence, any such payoffs can be sustained in a stationary equilibrium with effort \bar{a} . But then $\bar{a} < a^{FB}$, since if $\bar{a} = a^{FB}$, (DE) requires (7) to hold. So (DE) binds for all $\Pi \geq \bar{\Pi}$ and $U = \tilde{U}(\Pi)$. Since $\tilde{U}(\Pi) > 0$, there exists an interval of feasible $\Pi > \bar{\Pi}$. In particular, for the Π such that $\tilde{U}(\Pi) = 0$, $(1 - \delta) \frac{c(\bar{a})}{p} = \delta\Pi$. But $(1 - \delta) \frac{c(\bar{a})}{p} \geq \delta\bar{\Pi}$ and $\bar{\Pi} < \Pi$, obtaining contradiction.

Part 2: for any $\Pi < \bar{\Pi}$, $\Pi_L = \Pi < \Pi_H$. Denote the right-hand and left-hand derivatives of \tilde{U} by $\partial_+ \tilde{U}$ and $\partial_- \tilde{U}$, respectively, and note that $\partial_- \tilde{U}(\Pi) \geq \partial_+ \tilde{U}(\Pi)$ because \tilde{U} is concave.

We first claim that if $\Pi < \bar{\Pi}$, then $\partial_+ \tilde{U}(\Pi_L) \leq \partial_+ \tilde{U}(\Pi) \leq \partial_- \tilde{U}(\Pi) \leq \partial_- \tilde{U}(\Pi_L)$. Suppose $\partial_+ \tilde{U}(\Pi_L) > \partial_+ \tilde{U}(\Pi)$. Since $\Pi < \bar{\Pi}$, $\tilde{U}(\Pi) > 0$; therefore, we can perturb the equilibrium by increasing Π_L without violating (IC). Doing so increases Π at rate $(1-p)\delta$ and decreases U at rate $(1-p)\delta \partial_+ \tilde{U}(\Pi_L)$. But this perturbation remains an equilibrium and hence $\partial_+ \tilde{U}(\Pi) \geq \frac{(1-p)\delta \partial_+ \tilde{U}(\Pi_L)}{(1-p)\delta}$, which contradicts our assumption. Similarly, if $\partial_- \tilde{U}(\Pi_L) < \partial_- \tilde{U}(\Pi)$, then we can prove contradiction by decreasing Π_L , which does not violate any constraint.

Next, we argue that $\partial_+ \tilde{U}(\cdot)$ is strictly decreasing for all $\Pi < \bar{\Pi}$, so that $\partial_+ \tilde{U}(\Pi_L) \leq \partial_+ \tilde{U}(\Pi)$ and $\partial_- \tilde{U}(\Pi) \leq \partial_- \tilde{U}(\Pi_L)$ imply that $\Pi = \Pi_L$. Towards contradiction, suppose that $\tilde{U}(\cdot)$ is linear on an interval $\Pi^A < \Pi^B$. Let $(b^i, a^i, \Pi_L^i, \Pi_H^i)$ be the bonus, action, and continuation payoffs associated with Π^i , $i \in \{A, B\}$. Then for any $\alpha \in (0, 1)$, $b =$

$\alpha b^A + (1 - \alpha)b^B$, $a = \alpha a^A + (1 - \alpha)a^B$, $\Pi_L = \alpha \Pi_L^A + (1 - \alpha)\Pi_L^B$, and $\Pi_H = \alpha \Pi_H^A + (1 - \alpha)\Pi_H^B$ are an equilibrium. If $a^A \neq a^B$, then the worker's payoff from this convex combination is strictly larger than $\alpha \tilde{U}(\Pi^A) + (1 - \alpha)\tilde{U}(\Pi^B)$ because $c(\cdot)$ is strictly convex. Hence, $a^A = a^B$.

Now, (DE) must bind for any $\Pi < \bar{\Pi}$. Suppose it does not, so $a^A = a^B \geq a^{FB}$. Then the upper bound of (LL) binds, since otherwise we could increase b , which increases U while holding total surplus fixed and therefore contradicts $\Pi < \bar{\Pi}$. Moreover, $\Pi_H \leq \bar{\Pi}$, since otherwise we could decrease it without affecting total surplus, which similarly contradicts $\Pi < \bar{\Pi}$. But then

$$(1 - \delta)b = (1 - \delta)a \geq (1 - \delta)a^{FB} > \delta\bar{\Pi} \geq \delta\Pi_H,$$

where the equality follows from binding (LL), the first weak inequality follows from $a \geq a^{FB}$, the strict inequality follows by (7), and the second weak inequality follows because $\Pi_H \leq \bar{\Pi}$. But then (DE) is violated; contradiction.

Since (DE) binds,

$$\Pi^A = (1 - \delta)y + \delta(1 - p)\Pi_L^A.$$

Furthermore, $\Pi_L^A \geq \Pi^A$, since if $\Pi_L^A < \Pi^A$ then $\partial_+\tilde{U}(\Pi_L^A) > \partial_+\tilde{U}(\Pi^A)$, since Π^A is the left endpoint of the linear segment. By an analogous argument, $\Pi_L^B \leq \Pi^B$.

If $\Pi_L^B < \Pi^B$, we can increase Π_L^B , which contradicts that Π^B is the right endpoint of the linear segment. So $\Pi_L^B = \Pi^B$, and by a similar argument, $\Pi_L^A = \Pi^A$. But then

$$\Pi^A = \frac{1 - \delta}{1 - \delta(1 - p)}a^A = \frac{1 - \delta}{1 - \delta(1 - p)}a^B = \Pi^B,$$

contradicting $\Pi^A < \Pi^B$.

We have shown that $\partial_+\tilde{U}(\cdot)$ is strictly decreasing for all $\Pi < \bar{\Pi}$. Therefore, $\Pi_L = \Pi$ on this range.

Part 3: effort, profit, and repayment dynamics. We argue that in the equilibrium yielding payoffs $(\tilde{U}(\Pi), \Pi) \in E$, $a = a^*(\Pi)$ as defined in the statement of Proposition 3. Indeed, suppose (7) holds. Then $a^*(\Pi) = a^{FB}$ for any $\Pi \geq \bar{\Pi}$ and so $a = a^*(\Pi)$ follows immediately. Otherwise, we argued in Part 2 that (DE) binds. Therefore,

$$\Pi = (1 - \delta)pa + \delta(1 - p)\Pi_L,$$

Since $\Pi = \Pi_L < \bar{\Pi}$, $a = \frac{1-\delta(1-p)}{(1-\delta)p}\Pi < a^{FB}$ and so $a = a^*(\Pi)$.

For profit dynamics, we argue that $\Pi_H > \Pi$ whenever $\Pi < \bar{\Pi}$. Suppose not; then $\Pi_H < \bar{\Pi}$. But then the upper bound of (LL) must bind. Otherwise, we could increase b and increase Π_H so that $(1 - \delta)b + \delta\Pi_H$ is constant, which satisfies all constraints because $\tilde{U}(\Pi_H) + \Pi_H$ is strictly increasing in this range. Therefore,

$$\Pi = \delta p\Pi_H + \delta(1 - p)\Pi_L \leq \delta\Pi,$$

since $\Pi_H \leq \Pi$ by assumption and $\Pi_L = \Pi$ by Part 2.

Finally, we argue that a_t converges to a_{\max} as $t \rightarrow \infty$ with probability 1 in any profit-maximizing equilibrium. Since $\Pi_L = \Pi$ and high output is realized with probability $p > 0$ in each period, it suffices to show that with probability 1, any profit-maximizing equilibrium reaches a period in which $\Pi_H \geq \bar{\Pi}$. So long as $\Pi_H < \bar{\Pi}$, (LL) binds and so

$$\Pi_H - \Pi = \frac{1 - \delta}{\delta p}\Pi.$$

For any $K > 0$, $\Pi > 0$ in the first period of any equilibrium in which the project is funded. Therefore, the manager's profit increases by an amount that is bounded away from 0 every time high output is realized (and remains constant following low output). So $\Pi_H \geq \bar{\Pi}$ after a finite number of high outputs, which happen with probability 1 as $t \rightarrow \infty$. ■

A.5 Proof of Corollary 1

Fastest Repayment Path: We first claim that this payment path repays the creditor as quickly as possible. Indeed, for any payment path such that $b > 0$ in a period for which $K(U_H, \Pi_H) > 0$, we can decrease b and increase U_H so that $(1 - \delta)b + \delta U_H$ remains constant, and increase r so that $b + r$ remains constant. Lemma 1 implies that both worker and creditor earn the same expected surplus following this perturbation. However, perturbing the equilibrium in this way decreases $K(U_H, \Pi_H)$ and therefore repays the creditor faster. Since both (DE) and the upper bound of (LL) bind whenever $\Pi_H < \bar{\Pi}$ in any profit-maximizing equilibrium, the specified payment path is consistent with a profit-maximizing equilibrium.

Slowest Repayment Path: This payment path sets $U = 0$ in each period, which clearly maximizes $K(U, \Pi) = \tilde{U}(\Pi) - U$ in that period and so repays the creditor as slowly as possible. The proof of Proposition 3 shows that both (DE) and the upper bound of (LL) bind whenever $\Pi_H < \bar{\Pi}$. This payment path also satisfies the lower bound of (LL), since $b = \frac{c(a)}{p} \geq 0$ and $r = a - b \geq 0$ because $pa - c(a) \geq 0$ for any $a \leq a_{max}$. Therefore, these payments are consistent with a profit-maximizing equilibrium. ■

A.6 Proof of Proposition 4

Analogous to the set E , define

$$E^T \equiv \{(U, \Pi) | \exists K \geq 0 \text{ such that } (U, \Pi, K) \text{ are truth-telling equilibrium payoffs}\}.$$

Define the problem (P) as maximizing (1) subject to (PK-A)-(LL), $(U_H, \Pi_H) \in E^T$ and $(U_L, \Pi_L) \in E^T$, and (TT), with the restriction to $a \in \{0, y\}$. Let $K^T(U, \Pi)$ be the value function for (P).

This proof characterizes $K^T(\cdot)$ with a series of lemmas, then uses that characterization to prove Proposition 4.

Lemma 2 Define

$$E_1 \equiv \{(U, \Pi) \in E^T | \exists \text{ a solution to (P) with } a = y\} \subseteq E^T,$$

and let $E_0 \equiv E^T \setminus E_1$. Then $(0, 0) \in E_0$ and $K^T(0, 0) = 0$, so that $(0, 0)$ can be supported by liquidating the firm. Moreover, any $(U, \Pi) \in E_0$ can be implemented by randomizing between $(0, 0)$ and some $(U', \Pi') \in E_1$.

Proof of Lemma 2

First, note that $(0, 0) \in E_0$, since otherwise (PK-P) would be violated. Consider any (U, Π) that can be implemented with $a = 0$. Then $U_L = \frac{U}{\delta}$, $\Pi_L = \frac{\Pi}{\delta}$, and consequently $K^T(U, \Pi) = \delta K^T\left(\frac{U}{\delta}, \frac{\Pi}{\delta}\right)$. In particular, $K^T(0, 0) = \delta K^T(0, 0)$ and so $K^T(0, 0) = 0$, which can be attained through liquidation.

Consider $(U, \Pi) \in E_0$ with $(U, \Pi) \neq (0, 0)$. If (U, Π) can be implemented with $a = 0$, then $K^T(U, \Pi) = \delta K^T\left(\frac{U}{\delta}, \frac{\Pi}{\delta}\right) = \delta K^T\left(\frac{U}{\delta}, \frac{\Pi}{\delta}\right) + (1 - \delta)K^T(0, 0) \leq K^T(U, \Pi)$, where the first

equality follows by the argument above, the second follows because $K^T(0, 0) = 0$, and the inequality holds because K^T is concave. So $K^T(U, \Pi)$ is linear between $(0, 0)$ and $(\frac{U}{\delta}, \frac{\Pi}{\delta})$ for any $(U, \Pi) \in E_0$. Let (U', Π') be the right endpoint of this linear segment, and note that $(U', \Pi') \in E_1$. Therefore, any $(U, \Pi) \in E_0$ can be implemented by randomizing between liquidation and some $(U', \Pi') \in E_1$.

Finally, if $(U, \Pi) \in E_0$ can be implemented with a such that $0 < \Pr\{a = y\} < 1$, then (U, Π) can also be implemented by randomizing between a point in E_1 and a point that can be implemented with $a = 0$. So by the above argument, such (U, Π) can also be implemented by randomizing between continuation and liquidation. ■

Now, define

$$\begin{aligned}\Pi_{\max} &= py - c; \\ \Pi_f &= \frac{p(1-\delta)y}{1-(1-p)\delta}; \\ \tilde{U}^T(\Pi) &= \max \{U | (\Pi, U) \in E^T\}.\end{aligned}$$

Note that (4) implies that $\Pi_f \leq \Pi_{\max}$.

Lemma 3 *For any $(U, \Pi) \in E^T$,*

1. $U + \Pi = \Pi_{\max}$ if and only if $\Pi \in [\Pi_f, \Pi_{\max}]$ and $U = \tilde{U}^T(\Pi)$;
2. If $U + \Pi < \Pi_{\max}$, then $K^T(U, \Pi) + U + \Pi < py - c$.

Proof of Lemma 3

Suppose that $U + \Pi = \Pi_{\max}$. Then Π_{\max} is the maximum feasible total surplus, so $K^T(U, \Pi) = 0$ and hence $U = \tilde{U}^T(\Pi)$. Define Π'_f as the manager's smallest equilibrium payoff such that $\tilde{U}^T(\Pi'_f) + \Pi'_f = \Pi_{\max}$, and denote Π_H and Π_L as the associated continuation profits. Note that $a = y$ for $(\tilde{U}^T(\Pi'_f), \Pi'_f)$, and so

$$\begin{aligned}\Pi'_f &= p((1-\delta)(y-b) + \delta\Pi_H) + (1-p)\delta\Pi_L \\ &\geq p(1-\delta)y + (1-p)\delta\Pi'_f,\end{aligned}$$

where the equality holds by (PK-A) and the inequality follows because (DE) implies $\delta\Pi_H \geq (1-\delta)b$, and $\Pi_L \geq \Pi'_f$ in order for sum of the manager's and worker's payoffs to equal Π_{\max} . Rearranging this expression yields $\Pi'_f \geq \Pi_f$.

Now, suppose $\Pi \geq \Pi_f$, and consider the set of stationary strategies such that $a = 1$, $r = 0$ and $b \in \left[\frac{c}{p}, \frac{\delta\Pi_f}{1-\delta} \right]$. It is straightforward to argue that all of these payments can be sustained in a relational contract. With $b = \frac{c}{p}$, the manager earns Π_{\max} ; with $b = \frac{\delta\Pi_f}{1-\delta}$, the manager's payoff is Π_f . Therefore, $\tilde{U}^T(\Pi) + \Pi = py - c$ for any $\Pi \geq \Pi_f$. Combined with the result that $\Pi'_f \geq \Pi_f$, we conclude that $\Pi'_f = \Pi_f$ and that for any $\Pi \in [\Pi_f, \Pi_{\max}]$, $\tilde{U}^T(\Pi) + \Pi = py - c$, which proves part 1.

Next, define

$$z \equiv \min \{ U + \Pi | K^T(U, \Pi) + U + \Pi = \Pi_{\max} \} .$$

Suppose $z < \Pi_{\max}$, and choose $(U, \Pi) \in E^T$ such that $U + \Pi = z$ and $K^T(U, \Pi) + U + \Pi = py - c$. Then it must be that $a = 1$ with probability 1, and moreover $U_L + \Pi_L + K^T(U_L, \Pi_L) = py - c$. Then summing (PK-A) and (PK-P) implies that

$$\begin{aligned} z &= (1 - \delta)(py - c - pr) + \delta p(U_H + \Pi_H - U_L - \Pi_L) + \delta(U_L + \Pi_L) \\ &\geq (1 - \delta)(py - c) + \delta(U_L + \Pi_L), \end{aligned}$$

where the inequality follows from (TT). Since $z < py - c$, $U_L + \Pi_L < z$. But $U_L + \Pi_L + K^T(U_L + \Pi_L) = py - c$, yielding a contradiction. ■

Lemma 4 *The following hold:*

1. For $\Pi \leq \Pi_f$,

$$\tilde{U}^T(\Pi) = \frac{\tilde{U}^T(\Pi_f)}{\Pi_f} \Pi.$$

2. For any $\Pi \in [0, \Pi_{\max}]$, $K^T(\tilde{U}^T(\Pi), \Pi) = 0$.

3. For any (U, Π) with $U + \Pi < py - c$, $K^T(U, \Pi) + \Pi$ is strictly increasing in Π . For $\Pi \geq \Pi_f$, $K^T(U, \Pi) + U$ is strictly increasing in U .

Proof of Lemma 4

Part 1: Define $\xi = \max \left\{ \frac{U}{\Pi} | (U, \Pi) \in E^T \right\}$ and let (U, Π) be such that $\frac{U}{\Pi} = \xi$. Lemma 2 implies that we can take $(U, \Pi) \in E_1$, and it is immediate from the definition of $\tilde{U}^T(\cdot)$ that we can take $U = \tilde{U}^T(\Pi)$. We claim that $\Pi = \Pi_f$. For any $\Pi > \Pi_f$, $\tilde{U}^T(\Pi) + \Pi = \Pi_{\max}$, and so $\tilde{U}^T(\Pi)$ is strictly decreasing in Π on this range. So $\frac{\tilde{U}^T(\Pi_f)}{\Pi_f} > \frac{\tilde{U}^T(\Pi)}{\Pi}$ for any $\Pi > \Pi_f$.

Consider $\Pi < \Pi_f$. To show this, we note two properties. First, $r = 0$ in any equilibrium with payoffs $(\tilde{U}^T(\Pi), \Pi)$, since otherwise we could decrease r and increase b so that $r + b$ is constant and the worker earns a strictly higher payoff than $\tilde{U}^T(\Pi)$.

Second, we claim that there exists an equilibrium giving $(\tilde{U}^T(\Pi), \Pi)$ in which $\Pi_H > \Pi_f$. Recall that $(\tilde{U}^T(\Pi), \Pi) \in E_1$, and note that either (DE) or the upper bound of (LL) must bind, since otherwise we could increase b and hence increase the worker's payoff. Suppose the upper bound of (LL) is slack, so $b < y$. From the public randomization device, $\tilde{U}^T(\Pi)$ is concave, so $\Pi + \tilde{U}^T(\Pi)$ is increasing in Π (because it is constant for $\Pi \geq \Pi_f$). Therefore, consider increasing b and $\frac{\delta}{1-\delta}\Pi_H$ by the same amount. Doing so holds the manager's payoff constant and gives the worker a higher payoff. So there exists an equilibrium giving $(\tilde{U}^T(\Pi), \Pi)$ in which $b = y$. But then

$$\delta\Pi_H \geq (1 - \delta)y > \frac{\delta p(1 - \delta)y}{1 - (1 - p)\delta} = \delta\Pi_f,$$

as desired.

Given these two properties,

$$\begin{aligned} 1 + \xi &= \frac{U + \Pi}{\Pi} \\ &= \frac{(1 - \delta)(py - c) + \delta p(py - c) + \delta(1 - p)(U_L + \Pi_L)}{\Pi} \\ &\leq \frac{(1 - \delta + \delta p)(py - c) + (1 - p)\Pi_L(1 + \xi)}{\Pi} \\ &\leq \frac{(1 - \delta + \delta p)(py - c) + (1 - p)\Pi_L(1 + \xi)}{(1 - \delta)py + \delta(1 - p)\Pi_L} \\ &\leq \frac{(1 - \delta + \delta p)(py - c)}{\tilde{U}^T(\Pi_f) + \Pi_f} \\ &= \frac{(1 - \delta)py}{\tilde{U}^T(\Pi_f) + \Pi_f} \end{aligned} \tag{8}$$

Here, the first equality follows from $\Pi_H > \Pi_f$ and so $\tilde{U}^T(\Pi_H) + \Pi_H = py - c$, the first inequality holds because $\frac{U_L + \Pi_L}{\Pi_L} \leq 1 + \xi$ by definition of ξ , the second inequality follows because (DE) implies that $\Pi \geq (1 - \delta)py + \delta(1 - p)\Pi_L$, the third inequality holds because $\delta(1 - p)\Pi_L \geq 0$, and the final equality holds by definition of Π_f and because $\tilde{U}^T(\Pi_f) + \Pi_f = py - c$.

We conclude that $\frac{\tilde{U}^T(\Pi_f)}{\Pi_f} = \xi$, as desired, which implies part 1 of Lemma 4 because $\tilde{U}^T(\Pi)$ is concave and so $\frac{\tilde{U}^T(\Pi)}{\Pi}$ is decreasing in Π , and strictly so unless $\tilde{U}^T(\cdot)$ is linear.

Part 2: Note that for $\Pi \geq \Pi_f$, Lemma 3 implies that $K^T(\tilde{U}^T(\Pi), \Pi) = 0$. For $\Pi < \Pi_f$, (8) holds with equality only if $\frac{U_L}{\Pi_L} = \xi$. But then $\Pi_L \geq \Pi_f$, implying that $K^T(U_L, \Pi_L) = 0$, and similarly $\Pi_H \geq \Pi_f$ so $K^T(U_H, \Pi_H) = 0$. Since $r = 0$ as well, $K^T(\tilde{U}^T(\Pi), \Pi) = 0$ in this range too.

Part 3: Lemma 3 and the concavity of $K^T(\cdot)$ imply that $K^T(U, \Pi) + U$ is strictly increasing in U for $\Pi \geq \Pi_f$. Similarly, Lemma 3, concavity of K^T , and the fact that $\tilde{U}^T(\Pi)$ is maximized at Π_f imply that $K^T(U, \Pi) + \Pi$ is strictly increasing whenever $U + \Pi < py - c$. ■

Given this characterization, we are prepared to prove our main result.

Proof of Proposition 4

Part 1: Suppose that $\Pi \geq \frac{p(1-\delta)y}{1-(1-p)\delta}$ and $K^T(U, \Pi) > 0$. Since $U + \Pi + K^T(U, \Pi) \leq \Pi_{max} \equiv py - c$, $U + \Pi < \Pi_{max} \equiv py - c$. Part 2 of Lemma 3 therefore implies that

$$K^T(U, \Pi) + U + \Pi < py - c.$$

Now, part 1 of Lemma 3 implies that $K^T(\tilde{U}^T(\Pi), \Pi) + \tilde{U}^T(\Pi) + \Pi = py - c$, where $\tilde{U}^T(\Pi) > U$ by definition. Since

$$K^T(\tilde{U}^T(\Pi), \Pi) + \tilde{U}^T(\Pi) + \Pi > K^T(U, \Pi) + U + \Pi$$

and $K^T(\cdot)$ is weakly concave, we conclude that

$$K^T(U', \Pi) + U' + \Pi > K^T(U, \Pi) + U + \Pi$$

for any $U' > U$.

Part 2: Suppose $U_H + \Pi_H < py - c$. First, we argue that $r + b = y$ whenever $U_H + \Pi_H < py - c$. Note that $\Pi_H < \Pi_f$. Suppose $r + b < y$, and consider an alternative that increases r by $\epsilon > 0$ and increases Π_H by $\frac{1-\delta}{\delta}\epsilon$. For $\epsilon > 0$ sufficiently small, this perturbation is feasible—in particular, Lemma 4 implies $(U_H, \Pi_H + \frac{1-\delta}{\delta}\epsilon) \in E^T$ because $\Pi_H < \Pi_f$ —and it

continues to satisfy the constraints of (P). Moreover,

$$\delta K^T \left(U_H, \Pi_H + \frac{1-\delta}{\delta} \epsilon \right) + (1-\delta)\epsilon > \delta K^T(U_H, \Pi_H)$$

by part 3 of Lemma 4. So the original equilibrium cannot be on the frontier defined by $K^T(\cdot)$; contradiction.

Next, we show that $b = 0$. Suppose $b > 0$, and consider increasing r and decreasing b by $\epsilon > 0$, and increasing U_H by $\frac{1-\delta}{\delta}\epsilon$. As before, this perturbation satisfies the constraints of (P). It is also feasible for sufficiently small $\epsilon > 0$, since $\Pi_H > \Pi_f$ from the proof of Lemma 4 and so $U_H < py - c - \Pi_H \leq \tilde{U}^T(\Pi_H)$.

Now, since $\Pi_H > \Pi_f$,

$$\delta\epsilon + \delta K \left(U_H + \frac{1-\delta}{\delta} \epsilon, \Pi_H \right) > \delta K(U_H, \Pi_H)$$

by part 3 of Lemma 4. So the creditor earns a strictly higher payoff in the perturbed equilibrium. Hence, $b = 0$ and $r = y$.

Part 3: By Lemma 2, it suffices to show that when $K(U, \Pi) > 0$, continuation play at some successor history lies in E_0 with positive probability. Suppose not; then $K(U, \Pi) + U + \Pi = py - c$. But then Lemma 3 (part 1) and Lemma 4 (part 2) imply that $K(U, \Pi) = 0$; contradiction. ■

B Details of Empirical Analysis

B.1 Sample Construction

We access the Amadeus database using WRDS. Since the required variables are often unavailable for small firms, we include only large and very large companies. A firm is considered large or very large if it meets at least one of the following conditions: more than 150 employees, operating revenue higher than 10 mln EUR or total assets of 20 mln EUR or more. We use the entire available time period, 1985-2017, but since Amadeus provides no more than 10 years of data for each firm, for most of firms our data comes from 2006/07-2016/17. We keep the consolidated data whenever both consolidated and unconsolidated data are available. If a firm has more than one observation per year, we use the latest.

Capital is defined as the log of fixed assets and labor is defined as the log of cost of employees. We use operating revenue as a proxy for output, since sales entails more missing data. Our proxy for intermediate inputs is the log of material costs, and Industry is given by 2-digits codes from the NACE2 classification. We keep only manufacturing firms, which have NACE2 codes between 10 and 32, and we drop firms that have fewer than 5 yearly observations of capital. To reliably estimate TFP by (industry X country), we keep only those industry-country pairs that have at least 1000 firm-year observations with non-missing capital, labor, output and intermediate inputs measures. Our results with OLS-based TFP-R are similar if we use a threshold of 100 observations instead; our other measures, however, are more data-intensive and so frequently fail to converge if the number of observations is too small.

This restriction greatly reduces our working sample for two reasons. First, not all information is available for all countries. For example, firms in the United Kingdom do not report intermediate outputs and so are dropped. Second, smaller countries or industries might not have 1000 observations. Nevertheless, all but one industry (tobacco manufacturing) is included for at least one country, and all large European countries, except for the United Kingdom, are included in the final sample. Our final data has 127,703 observations. To make sure our results are not driven by outliers, we trim both regressands and regressors at the 5th and 95th percentile. Table 5 presents summary statistics for our final sample.

Table 5: Summary Statistics for Estimation Sample

| | Mean | Median | Std Dev | Min | Max |
|-------------------------------|---------|---------|---------|---------|--------|
| Leverage | 0.140 | 0.114 | 0.112 | 0 | 0.553 |
| Δ Leverage | -0.0024 | -0.0024 | 0.0416 | -0.1314 | 0.1472 |
| Operating revenue (mln Euro) | 60.0 | 25.3 | 125.7 | 0.5 | 8277.8 |
| Fixed assets (Mln Euro) | 16.1 | 6.4 | 27.5 | 0.3 | 223.5 |
| Costs of employees (Mln Euro) | 9.7 | 4.1 | 26.6 | 0.0 | 4115.3 |
| Number of employees | 140.18 | 90 | 135.36 | 15 | 775 |
| Material costs (Mln Euro) | 33.3 | 12.7 | 76.5 | 0.0 | 2733.5 |
| Total assets (Mln Euro) | 45.7 | 21.7 | 78.2 | 0.4 | 2756.6 |
| TFP-R (OLS) | -0.059 | -0.096 | 0.420 | -0.903 | 1.099 |
| Δ TFP-R (OLS) | -0.004 | -0.005 | 0.147 | -1.477 | 1.771 |
| TFP-R (LP) | 2.792 | 2.343 | 3.448 | -4.109 | 11.681 |
| Δ TFP-R (LP) | 0.004 | 0.003 | 0.111 | -3.485 | 3.609 |
| TFP-R (GNR) | 23.348 | 23.343 | 0.676 | 15.824 | 26.043 |
| Δ TFP-R (GNR) | 0.021 | 0.017 | 0.189 | -5.363 | 5.524 |
| Observations | 127703 | | | | |

B.2 TFP Calculations

We use three methods to calculate revenue-based TFP. Our first approach runs a linear regression of log output on log capital and log labor. We run this regression separately for each country-industry pair in our final sample, compute coefficients of capital and labor, and save the residuals as TFP measure. This method of estimating TFP is potentially biased, since firms might adjust their inputs in anticipation of productivity shocks. Two well-known proxy methods address this problem: Olley and Pakes (1996) and Levinsohn and Petrin (2003), which respectively use investment or intermediate inputs to identify the coefficient on labor, and then back out TFP using this estimate. Since Amadeus does not consistently report investment, we use Levinsohn and Petrin (2003) (henceforth LP) and proxy intermediate inputs using material costs. These calculations are performed using the *levpet* function in Stata. As a further test, we adopt the methods of Gandhi et al. (2017) (henceforth GNR), which address some of the weaknesses in these TFP measures.¹⁰ We gratefully acknowledge help from the authors of that paper, who provided the Stata code for our estimation.

¹⁰We use this method rather than the alternative proposed by Ackerberg et al. (2015) because it is better suited to TFP calculations for a gross output production function.

B.3 Including Lagged Productivity

Table 6: Changes in TFP and Changes in Leverage - Including Lag of Productivity

| | (1) $\Delta\text{TFP-}$ OLS | (2) $\Delta\text{TFP-}$ OLS | (3) $\Delta\text{TFP-LP}$ | (4) $\Delta\text{TFP-LP}$ | (5) $\Delta\text{TFP-}$ GNR | (6) $\Delta\text{TFP-}$ GNR |
|--------------------------------|-----------------------------------|-----------------------------------|------------------------------|------------------------------|-----------------------------------|-----------------------------------|
| $\Delta\text{Leverage}$ | -0.427 (0.0470) | -0.415 (0.0416) | -0.160 (0.0225) | -0.152 (0.0194) | -0.459 (0.0731) | -0.400 (0.0644) |
| $\Delta\text{Leverage (t-1)}$ | -0.198 (0.0322) | -0.144 (0.0240) | -0.0891 (0.0194) | -0.0723 (0.0195) | -0.205 (0.0434) | -0.123 (0.0271) |
| $\Delta\text{Dep. Var. (t-1)}$ | -0.247 (0.0136) | -0.155 (0.0183) | -0.291 (0.0121) | -0.186 (0.0254) | -0.267 (0.0235) | -0.161 (0.0288) |
| N | 123818 | 123818 | 123818 | 123818 | 123811 | 123811 |
| SD of Dep Var | 0.12 | 0.12 | 0.08 | 0.08 | 0.16 | 0.16 |
| SD of ΔLev | 0.044 | 0.044 | 0.044 | 0.044 | 0.044 | 0.044 |
| Clustering | Industry | Industry | Industry | Industry | Industry | Industry |
| Firm FE | Yes | - | Yes | - | Yes | - |
| Year FE | Yes | - | Yes | - | Yes | - |
| Industry \times Year FE | - | Yes | - | Yes | - | Yes |

Additional controls include total fixed assets and number of employees. Leverage is measured as the ratio of non-current liabilities to total assets (book value). Dependent variable is change in TFP-R which is calculated based on OLS specification (columns 1 and 3), Levinsohn and Petrin (2003) method (LP, columns 2 and 4) and Gandhi et al. (2017) method (GNR, columns 3 and 6). In the bottom rows, standard deviation of dependent variable and of change in leverage is a within-firm value and median across all firms is presented. Standard errors are clustered on the industry level and shown in the parentheses.

B.4 Magnitudes Calculations

The most conservative estimate of the effect of leverage comes from the LP measure of TFP. The standard deviation of log-change in leverage in our final sample is 0.044, while the within-firm standard deviation of TFP changes for the median firm equals 0.081 (with a mean of 0.133). Using our estimated coefficient on leverage, we conclude that a one standard deviation increase in leverage is correlated with a decrease in TFP of 0.007, which equals 8.7% of the median standard deviation.

The other specifications yield larger estimates for this correlation. For example, the GNR

measure of TFP has coefficient of -0.475, with a median within-firm standard deviation in TFP of 0.156. The implied magnitude of the effect of a one standard deviation increase in leverage is therefore 13.3% of the median standard deviation in TFP. For our OLS measure of TFP, the magnitude is 15.3%. From Table 2, the effects of the previous year's leverage on productivity are about one quarter the size of these estimates.